Solve Rational Equations

1. Solve the following equation. Remember to check for extraneous solutions.

$$\frac{1}{x-6}+\frac{x}{x-2}=\frac{4}{x^2-8x+12}$$

2. Find all solutions to the following equation. If there are any extraneous solutions, identify them and explain why they are extraneous.

$$\frac{7}{b+3}+\frac{5}{b-3}=\frac{10b}{b^2-9}$$

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$$\left(\frac{1}{x-6}+\frac{x}{x-2}\right)(x-6)(x-2)=\left(\frac{4}{(x-6)(x-2)}\right)(x-6)(x-2)$$
$$(x-2)+x(x-6)=4$$
$$x^2-6x+x-2=4$$
$$x^2-5x-6=0$$
$$(x-6)(x+1)=0$$

The solutions are 6 and -1.

Because x is not allowed to be 6 in order to avoid division by zero, the solution 6 is extraneous; thus, -1 is the only solution to the given rational equation.

2. Find all solutions to the following equation. If there are any extraneous solutions, identify them and explain why they are extraneous.

$$\frac{7}{b+3}+\frac{5}{b-3}=\frac{10b}{b^2-9}$$

First, note that we must have $x \neq 3$ and $x \neq -3$.

Using the equating numerators method: $\frac{7(b-3)}{(b-3)(b+3)}+\frac{5(b+3)}{(b-3)(b+3)}=\frac{10b}{(b-3)(b+3)}$

Matching numerators, we have $7b-21+5b+15=10b$, which leads to $12b-6=10b$; therefore, $b=3$.

However, since the excluded values are 3 and -3, the solution 3 is an extraneous solution, and there is no solution to $\frac{7}{b+3}+\frac{5}{b-3}=\frac{10b}{b^2-9}$.