Lesson 8: Applying the Properties of Operations to Add and Subtract Rational Numbers

Classwork

Example 1: The Opposite of a Sum is the Sum of its Opposites

Explain the meaning of: *"The opposite of a sum is the sum of its opposites."* Use a specific math example.

Rational Number	Rational Number	Sum	Opposite of the Sum

Opposite Rational Number	Opposite Rational Number	Sum

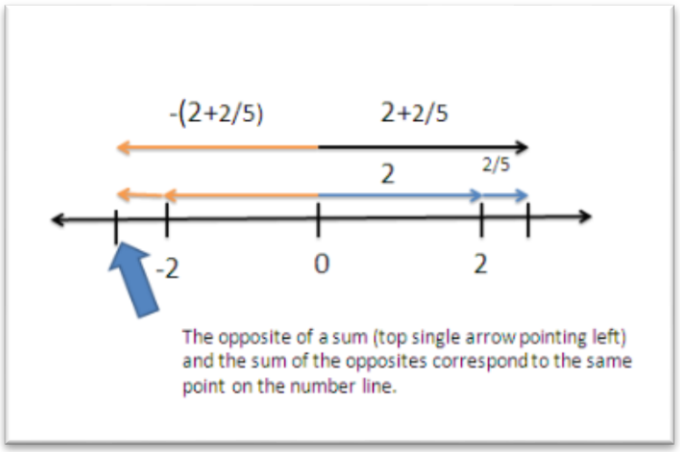
Exercise 1

Represent the following expression with a single rational number.

$-2\frac{2}{5}+3\frac{1}{4}-\frac{3}{5}$

Example 2: A Mixed Number is a Sum

Use the number line model shown below to explain and write the opposite of $2\frac{2}{5}$ as a sum of two rational numbers.



Exercise 2

Rewrite each mixed number as the sum of two signed numbers.

a. $-9\frac{5}{8}$

b. $-2\frac{1}{2}$

c. $8\frac{11}{12}$

Exercise 3

Represent each sum as a mixed number.

a. $-1 + \left(-\frac{5}{12}\right)$

b. $30 + \frac{1}{8}$

c. $-17 + \left(-\frac{1}{9}\right)$

Exercise 4

Mr. Mitchell lost 10 pounds over the summer by jogging each week. By winter time, he had gained $5\frac{1}{8}$ pounds. Represent this situation with an expression involving signed numbers. What is the overall change in Mr. Mitchell's weight?

Exercise 5

Jamal is completing a math problem and represents the expression $-5\frac{5}{7}+8-3\frac{2}{7}$ with a single rational number as shown in the steps below. Justify each of Jamal's steps. Then, show another way to solve the problem.

$$= -5\frac{5}{7}+8+\left(-3\frac{2}{7}\right)$$

$$= -5\frac{5}{7}+\left(-3\frac{2}{7}\right)+8$$

$$= -5+\left(-\frac{5}{7}\right)+(-3)+\left(-\frac{2}{7}\right)+8$$

$$= -5+\left(-\frac{5}{7}\right)+\left(-\frac{2}{7}\right)+(-3)+8$$

$$= -5+(-1)+(-3)+8$$

$$= -6+(-3)+8$$

$$= (-9)+8$$

$$= -1$$

Lesson Summary

- Use the properties of operations to add and subtract rational numbers more efficiently. For instance:
  $-5\frac{2}{9} + 3.7 + 5\frac{2}{9} = \left(-5\frac{2}{9} + 5\frac{2}{9}\right) + 3.7 = 0 \ + 3.7 = 3.7$
- The opposite of a sum is the sum of its opposites as shown in the examples that follow:
  $-4\frac{4}{7} = -4 + \left(-\frac{4}{7}\right)$
  $-(5 \ + \ 3) \ = \ -5 \ + \ (-3)$

Problem Set

1. Represent each sum as a single rational number.
   a. $-14 + \left(-\frac{8}{9}\right)$
   b. $7 + \frac{1}{9}$
   c. $-3 + \left(-\frac{1}{6}\right)$

Rewrite each of the following to show that *the opposite of a sum is the sum of the opposites.* Problem 4 has been completed as an example.

2. $-(9 + 8) = \ -9 + (-8)$
   $-17 \quad = \quad -17$

3. $-\left(\frac{1}{4} + 6\right)$

4. $-(10 + (-6))$

5. $-\left((-55) + \frac{1}{2}\right)$

6. Meghan said the opposite of the sum of -12 and 4 is 8. Do you agree? Why or why not?

7. Jolene lost her wallet at the mall. It had $\$10$ in it. When she got home her brother felt sorry for her and gave her $\$5.75$. Represent this situation with an expression involving rational numbers. What is the overall change in the amount of money Jolene has?

8. Isaiah is completing a math problem and is at the last step: $25 - 28\frac{1}{5}$. What is the answer? Show your work.

9. A number added to its opposite equals zero. What do you suppose is true about *a sum added to its opposite*? Use the following examples to reach a conclusion. Express the answer to each example as a single rational number.
   a. $(3 + 4) + (-3 + -4)$
   b. $(-8 + 1) + (8 + (-1))$
   c. $\left(-\frac{1}{2} + \left(-\frac{1}{4}\right)\right) + \left(\frac{1}{2} + \frac{1}{4}\right)$