Lesson 1: Ratios

Student Outcomes

- Students understand that a *ratio* is an ordered pair of non-negative numbers, which are not both zero. Students understand that a ratio is often used instead of describing the first number as a multiple of the second.
- Students use the precise language and notation of ratios (e.g., $3:2$, 3 to 2). Students understand that the order of the pair of numbers in a ratio matters and that the description of the ratio relationship determines the correct order of the numbers. Students conceive of real-world contextual situations to match a given ratio.

Lesson Notes

The first two lessons of this module will develop the students' understanding of the term *ratio*. A ratio is always a pair of numbers, such as $2:3$ and never a pair of quantities such as 2 cm : 3 sec. Keeping this straight for students will require teachers to use the term ratio correctly and consistently.". Students will be required to separately keep track of the units in a word problem. To help distinguish between ratios and statements about quantities that define ratios, we use the term *ratio relationship* to describe a phrase in a word problem that indicates a ratio. Typical examples of ratio relationship descriptions include "3 cups to 4 cups," "5 miles in 4 hours," etc. The ratios for these ratio relationships are $3:4$ and $5:4$, respectively.

Classwork

Example 1 (15 minutes)

Read the example aloud.

> **Example 1**
>
> **The coed soccer team has four times as many boys on it as it has girls. We say the ratio of the number of boys to the number of girls on the team is $4:1$. We read this as "four to one."**

- Let's create a table to show how many boys and how many girls on are on the team.

Create a table like the one shown below to show possibilities of the number of boys and girls on the soccer team. Have students copy the table into their student packet.

# of Boys	# of Girls	Total # of Players
4	1	5

- So, we would have four boys and one girl on the team for a total of five players. Is this big enough for a team?
  - *Adult teams require 11 players, but youth teams may have fewer. There is no right or wrong answer; just encourage the reflection on the question, thereby connecting their math work back to the context.*
- What are some other options that show four times as many boys as girls or a ratio of boys to girls of 4 to 1?
  - *Have students add each option given to their table.*

# of Boys	# of Girls	Total # of Players
4	1	5
8	2	10
12	3	15

- From the table, we can see that there are four boys for every one girl on the team.

Read the example aloud.

Suppose the ratio of number of boys to number of girls on the team is 3: 2.

Create a table like the one shown below to show possibilities of the number of boys and girls on the soccer team. Have students copy the table into their student packets.

# of Boys	# of Girls	Total # of Players
3	2	5

- What are some other options that show that there are three boys for every two girls on the team?

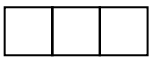
# of Boys	# of Girls	Total # of Players
3	2	5
6	4	10
9	6	15

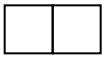
- I can't say there are 3 times as many boys as girls. What would my multiplicative value have to be? There are ____ as many boys as girls.

Encourage the students to articulate their thoughts, guiding them to say there are $\frac{3}{2}$ as many boys as girls.

- Can you visualize $\frac{3}{2}$ as many boys as girls?

- Can we make a tape diagram (or bar model) that shows that there are $\frac{3}{2}$ as many boys as girls?

Boys 

Girls 

- Which description makes the relationship easier to visualize: saying the ratio is 3 to 2 or saying there are 3 halves as many boys as girls?
  - *There is no right or wrong answer. Have students explain why they picked their choices.*

Example 2 (8 minutes): Class Ratios

Discussion (4 minutes)

Direct students:

- Find the ratio of boys to girls in our class.
- Raise your hand when you know: What is the ratio of boys to girls in our class?
- How can we say this as a multiplicative comparison without using ratios? Raise your hand when you know. *Allow for choral response when all hands are raised.*
- Write the ratio of number of boys to number of girls in your student packet under Example 2, Question 1.
- Compare your answer with your neighbor's answer. Does everyone's ratio look exactly the same?
  - Allow for discussion of differences in what students wrote. Communicate the following in the discussions:
    1. It is ok to use either the colon symbol or the word 'to' between the two numbers of the ratio.
    2. The ratio itself does not have units or descriptive words attached.
- Raise your hand when you know: What is the ratio of number of girls to number of boys in our class?
- Write the ratio down in your packet as number 2.
- Is the ratio of number of girls to number of boys the same as the ratio of number of boys to number of girls?
  - *Unless in this case there happens to be an equal number of boys and girls, then no, the ratios are not the same. Indicate that order matters.*
- Is this an interesting multiplicative comparison for this class? Is it worth commenting on in our class? If our class had 15 boys and 5 girls, might it be a more interesting observation?

For the exercise below, choose a way for students to indicate that they identify with the first statement (e.g., standing up or raising a hand). After each pair of statements below, have students create a ratio of the first statement to the second statement verbally, in writing, or both. Consider following each pair of statements with a discussion of whether it seems like an interesting ratio to discuss. Or alternatively, when you have finished all of these examples, ask students which ratio they found most interesting.

Students record a ratio for each of the examples you provide:

1. You traveled out of state this summer.
2. You did not travel out of state this summer.
3. You have at least one sibling.
4. You are an only child.
5. Your favorite class is math.
6. Your favorite class is not math.

Example 2: Class Ratios

Record a ratio for each of the examples the teacher provides.

1. *Answers will vary. One example is 12: 10.*
2. *Answers will vary. One example is 10: 12.*
3. *Answers will vary. One example is 7: 15.*
4. *Answers will vary. One example is 15: 7.*
5. *Answers will vary. One example is 11: 11.*
6. *Answers will vary. One example is 11: 11.*

Exercise 1 (2 minutes)

Have students look around the classroom to think of their own ratios. Have students create written ratio statements that represent their ratios in one of the summary forms.

Exercise 1

My own ratio compares *number of students wearing jeans* **to** *number of students not wearing jeans*.

My ratio 16: 6

Exercise 2 (10 minutes)

Students work with partners to write ratios in words that could be represented by each ratio given. Encourage students to be precise about the order in which the quantities are stated (emphasizing that order matters) and about the quantities being compared. That is, instead of saying the ratio of boys to girls, encourage them to say, the ratio of the number of boys to the number of girls. After students develop the capacity to be very precise about the quantities in the ratio, it is appropriate for them to abbreviate their communication in later lessons. Just be sure their abbreviations still accurately convey the meaning of the ratio in the correct order.

MP.

Exercise 2

Using words, describe a ratio that represents each ratio below.

a. 1 to 12 *for every one year, there are twelve months*

b. 12: 1 *for every twelve months, there is one year*

c. 2 to 5 *for every 2 days of non-school days in a week, there are five school days*

d. 5 to 2 *for every 5 female teachers I have, there are 2 male teachers*

e. 10: 2 *for every 10 toes, there are 2 feet*

f. 2: 10 *for every 2 problems I can finish, there are 10 minutes that pass*

After completion, invite sharing and explanations to the chosen answers.

Point out the difference between ratios like, "for every one year, there are twelve months" and ratios like, "for every 5 female teachers I have, there are 2 male teachers." The first type represents a constant relationship that will remain true as the number of years or months increases and the second one is somewhat arbitrary and will not remain true if the number of teachers increases.

Closing (5 minutes)

Provide students with this description:

A ratio is an ordered pair of non-negative numbers, which are not both zero. The ratio is denoted $A:B$ or A to B to indicate the order of the numbers. The number A is first, and the number B is second.

- What is a ratio? Can you verbally describe a ratio in your own words using this description?
- How do we write ratios?
  - *A colon B ($A:B$) or A 'to' B.*
- What are two quantities you would love to have in a ratio of $5:2$ but hate to have in a ratio of $2:5$?

Lesson Summary

A ratio is an ordered pair of non-negative numbers, which are not both zero.

The ratio is written $A:B$ or A *to* B to indicate the order of the numbers. The number A is first, and the number B is second.

The order of the numbers is important to the meaning of the ratio. Switching the numbers changes the relationship. The description of the ratio relationship tells us the correct order for the numbers in the ratio.

Exit Ticket (5 minutes)

Name _____ Date_____

Lesson 1: Ratios

Exit Ticket

1. Write a ratio for the following description: Kaleel made three times as many baskets as John during basketball practice.

2. Describe a situation that could be modeled with the ratio $4:1$.

3. Write a ratio for the following description: For every 6 cups of flour in a bread recipe, there are 2 cups of milk.

Exit Ticket Sample Solutions

The following solutions indicate an understanding of the objectives of this lesson:

1. **Write a ratio for the following description: Kaleel made three times as many baskets as John during basketball practice.**

   A ratio of $3:1$ or 3 to 1 can be used.

2. **Describe a situation that could be modeled with the ratio $4:1$.**

   Answers will vary but could include the following: For every four teaspoons of cream in a cup of tea, there is one teaspoon of honey.

3. **Write a ratio for the following description: For every 6 cups of flour in a bread recipe, there are 2 cups of milk.**

   A ratio of $6:2$ or 6 to 2 can be used, or students might recognize and suggest the equivalent ratio of $3:1$.

Problem Set Sample Solutions

1. **At the 6th grade school dance, there are 132 boys, 89 girls, and 14 adults.**
   a. **Write the ratio of number of boys to number of girls.**

      $132:89$ *(answers will vary)*

   b. **Write the same ratio using another form (A: B vs. A to B).**

      132 *to* 89 *(answers will vary)*

   c. **Write the ratio of number of boys to number of adults.**

      $132:14$ *(answers will vary)*

   d. **Write the same ratio using another form.**

      132 *to* 14 *(answers will vary)*

2. **In the cafeteria, 100 milk cartons were put out for breakfast. At the end of breakfast, 27 remained.**
   a. **What is the ratio of milk cartons taken to total milk cartons?**

      $73:100$ *(answers will vary)*

   b. **What is the ratio of milk cartons remaining to milk cartons taken?**

      $27:73$ *(answers will vary)*

3. Choose a situation that could be described by the following ratios, and write a sentence to describe the ratio in the context of the situation you chose.

   For example:

   $3:2$ When making pink paint, the art teacher uses the ratio $3:2$. For every 3 cups of white paint she uses in the mixture, she needs to use 2 cups of red paint.

   a. 1 to 2

      For every one cup of water, there are two half cups of water (answers will vary)

   b. 29 to 30

      For every 29 girls in the cafeteria, there are 30 boys (answers will vary)

   c. $52:12$

      For every 52 weeks in the year, there are 12 months (answers will vary)