

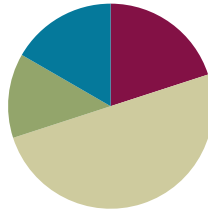
## Lesson 27

**Objective:** Explain equivalence by manipulating units and reasoning about their size.

**Related Topics:** [More Lesson Plans for the Common Core Math](#)

### Suggested Lesson Structure

■ Fluency Practice	(12 minutes)
■ Application Problem	(8 minutes)
■ Concept Development	(30 minutes)
■ Student Debrief	(10 minutes)
<b>Total Time</b>	<b>(60 minutes)</b>



### Fluency Practice (12 minutes)

- Sprint: Subtract by 7 **2.NBT.5** (8 minutes)
- Recognize the Fraction **3.G.2** (4 minutes)

### Sprint: Subtract by 7 (8 minutes)

Materials: (S) Subtract by 7 Sprint

### Recognize the Fraction (4 minutes)

Materials: (S) Personal white boards

T: (Project a shaded rectangular model.) This equals 1 whole. (Project 1 whole partitioned into 3 equal shaded units.) On your boards, write the fraction.

S: (Write  $\frac{3}{3}$ .)

T: (Project 2 wholes, each partitioned into 3 equal shaded units.) On your boards, write the fraction.

S: (Write  $\frac{6}{3}$ .)

T: (Project 3 wholes, each partitioned into 3 shaded parts.) On your boards, write the fraction.

S: (Write  $\frac{9}{3}$ .)

T: (Project 3 wholes, each partitioned into 3 parts. 3 parts in the first 2 wholes are shaded. 1 part of the third whole is shaded.) On your boards, write the fraction.



#### NOTES ON MULTIPLE MEANS OF REPRESENTATION:

Have ELLs “Recognize the Fraction” orally in order to practice speaking math language in English with the support of a model.

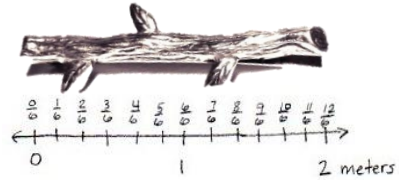
S: (Write  $\frac{7}{3}$ .)

Continue with possible sequence:  $\frac{4}{4}, \frac{8}{4}, \frac{12}{4}, \frac{9}{4}, \frac{6}{5}, \frac{9}{8}$ .

### Application Problem (8 minutes)

The branch of a tree is 2 meters long. Monica chops the branch for firewood. She cuts pieces that are  $\frac{1}{6}$  meter long. Draw a number line to show the total length of the branch. Partition and label each of Monica’s cuts.

- How many pieces does Monica have altogether?
- Write 2 equivalent fractions to describe the total length of Monica’s branch.



a) Monica has 12 pieces.

b)  $\frac{12}{6} = \frac{2}{1}$

### Concept Development (30 minutes)

Materials: (S) Template from Lesson 25, personal white boards, fraction strips

Pass out the template from Lesson 25 and have students slip it into their personal white boards.

T: Start on Side A of the template. Each rectangle represents 1 whole. Estimate to partition and label the rectangles. Divide each rectangle into thirds.

S: (Partition.)

T: How can we double the number of units in the second rectangle?

S: We cut each third in 2.

T: Go ahead and partition.

S: (Students partition.)

T: What’s our new unit?

S: Sixths!

Repeat this process for the third rectangle. Instead of having students double, have them triple the original thirds.

T: Label the fractions in each model.

S: (Label.)

**MP.3** T: What is different about these models?

S: They all started as thirds, but then we cut them into different parts. → The parts are different sizes.  
→ Yes, they’re different units.



#### NOTES ON MULTIPLE MEANS OF ENGAGEMENT:

For ELLs, demonstrate that words can have multiple meanings. Here, ‘cut’ means to draw a line (or lines) that divides the unit into smaller equal parts.

Students below grade level may benefit from revisiting the discussion of ‘doubling’, ‘tripling’, halving, and cutting unit fractions as presented in Lesson 22.

MP.3

- T: What is the same about these models?  
 S: The whole.  
 T: Talk to your partner about the relationship between the number of parts and the size of parts in each model.  
 S: 3 is the smallest number but thirds have the biggest size. → As I drew more lines to partition, the size of the parts got smaller. → That’s because the whole is cut into more pieces when there are ninths than when there are thirds.  
 T: (Each student has 3 fraction strips.) Fold all 3 fraction strips into halves.  
 S: (Students fold.)  
 T: Fold your second fraction strip to double the number of units.  
 S: We have to fold it again! (Students fold.)  
 T: What’s the new unit on your second fraction strip?  
 S: Fourths!  
 T: Fold your third fraction strip to double the number of units again.  
 S: (Students fold.)  
 T: Compare the number of parts and the size of the parts with the number of times you folded the strip. What happens to the size of the parts when you fold the strip more times?  
 S: The more I folded the smaller the parts got. → Yeah, that’s because you folded the whole to make more units.  
 T: Open your math journal to a new page and glue your strips in a column, making sure the ends line up. Glue them from the largest unit to the smallest.  
 S: (Students glue.)  
 T: Using your fraction strips find the fractions equivalent to  $\frac{4}{8}$ . Shade them.  
 S: (Students shade:  $\frac{4}{8}, \frac{2}{4}, \frac{1}{2}$ .)  
 T: Turn and talk with your partner. What do you notice about the size of parts and the number of parts in equivalent fractions?  
 S: You can see that there are more eighths than halves or fourths shaded to cover the same amount of the strip. → It’s the same as before then. As the number of parts gets bigger, the size of them gets smaller. → That’s because the shaded area in equivalent fractions doesn’t change even though the number of parts gets larger.

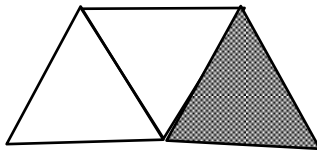
If necessary reinforce the concept with other examples using these fraction strips.

- T: Let’s practice this idea a bit more on our personal white boards. Draw my shape on your board. The entire figure represents 1 whole. (Show the image below.)



- S: (Students draw.)  
 T: Write the shaded fraction.  
 S: (Write  $\frac{1}{4}$ .)  
 T: Talk to your partner. How can you partition this shape to make an equivalent fraction with smaller units?  
 S: We can cut each small rectangle in 2 pieces from top to bottom to make eighths. → Or we can make 2 horizontal cuts to make twelfths.  
 T: Use one of these strategies now. (Circulate as students work to select a few different examples to share with the class.)  
 S: (Students partition.)  
 T: Let's look at our classmates' work. (Show examples in  $\frac{2}{8}, \frac{3}{12}, \frac{4}{16}$  etc.) As we partitioned with more parts, what happens to the shaded area and the number of parts needed to make them equivalent?  
 S: The size of the parts gets smaller, but the number of them gets larger.  
 T: Even though the parts changed, did the area covered by the shaded region change?  
 S: No.

You may want to have students practice independently. The following shape is more challenging because triangles are more difficult to make into equal parts.



**Problem Set (10 minutes)**

Students should do their personal best to complete the Problem Set within the allotted 10 minutes. For some classes, it may be appropriate to modify the assignment by specifying which problems they work on first. Some problems do not specify a method for solving. Students solve these problems using the RDW approach used for Application Problems.

**NOTES ON MULTIPLE MEANS OF ENGAGEMENT:**

Extend number 5 on the Problem Set for students above grade level. Instead of “doubling,” have students “triple” or “quadruple.” Let students choose the unit fraction into which the rectangle is partitioned.

Name: Gina Date: 4/5

1. Use the tape diagram to model equivalent fractions. Fill in the blanks and answer the following questions.

<p>4 sixths is equal to <u>2</u> thirds  <math>\frac{4}{6} = \frac{2}{3}</math></p> <p>The whole stays the same.</p> <p>What happened to the size of the equal parts when there were less equal parts?  <i>They got bigger.</i></p> <p>What happened to the number of equal parts when the equal parts became larger?  <i>There are less.</i></p>	<p>1 half is equal to <u>4</u> eighths  <math>\frac{1}{2} = \frac{4}{8}</math></p> <p>The whole stays the same.</p> <p>What happened to the size of the equal parts when there were more equal parts?  <i>They got smaller.</i></p> <p>What happened to the number of equal parts when the equal parts became smaller?  <i>You needed more.</i></p>
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2. 6 friends want to share three chocolate bars that are all the same size, represented by the 3 strips below. When the bars are unwrapped, the girls notice that the first chocolate bar is cut into 2 equal parts, the second is cut into 4 equal parts and the third is cut into 6 equal parts. How can the 6 friends share the chocolate bars equally, without breaking any of the pieces?

*#1* *#2* Will each friend get  $\frac{1}{2}$  of a candy bar. There are 6 halves, some candy bars just have more pieces but  $\frac{1}{2} = \frac{3}{6}$  and  $\frac{1}{2} = \frac{3}{6}$  so 1 bar can be shared between 2 friends.

### Student Debrief (10 minutes)

**Lesson Objective:** Explain equivalence by manipulating units and reasoning about their size.

The Student Debrief is intended to invite reflection and active processing of the total lesson experience.

Invite students to review their solutions for the Problem Set. They should check work by comparing answers with a partner before going over answers as a class. Look for misconceptions or misunderstandings that can be addressed in the Debrief. Guide students in a conversation to debrief the Problem Set and process the lesson.

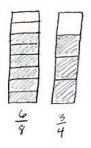
You may choose to use any combination of the questions below to lead the discussion.

- How did using the fraction strips help you with Problem 2? Talk about the relationship between them.
- What was your strategy for Problems 3 and 4? How did it change or stay the same?
- Why is it important that the magic wand in Problem 5 keeps the whole the same?
- How does the magic wand in Problem 5 make it easy to create equivalent fractions?

### Exit Ticket (3 minutes)

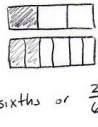
After the Student Debrief, instruct students to complete the Exit Ticket. A review of their work will help you assess the students' understanding of the concepts that were presented in the lesson today and plan more effectively for future lessons. You may read the questions aloud to the students.

3. When the whole is the same, why does it take 6 copies of 1 eighth to show 3 copies of 1 fourth? Draw a model to support your answer.



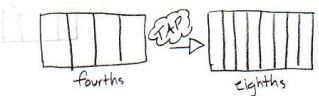
The unit "eighths" is smaller than the unit "fourths" so you need more copies of the smaller unit to be equal to the larger unit.

4. When the whole is the same, how many sixths does it take to make 1 third? Draw a model to support your answer.



2 sixths or  $\frac{2}{6} = \frac{1}{3}$

5. You have a magic wand that doubles the number of equal parts but keeps the whole the same size. Use your magic wand. In the space below draw to show what happens to a rectangle that is partitioned in fourths after you tap it with your wand. Use words and numbers to explain what happened.



First I started with fourths. That means 4 equal sized parts inside the whole. To double 4, I multiplied  $4 \times 2 = 8$ , so now the whole (after I tapped my magic wand) has 8 equal parts. The parts got smaller!

**A**

# Correct \_\_\_\_\_

Subtract.

1	$17 - 7 =$		23	$24 - 7 =$	
2	$7 - 7 =$		24	$34 - 7 =$	
3	$27 - 7 =$		25	$64 - 7 =$	
4	$8 - 7 =$		26	$84 - 7 =$	
5	$18 - 7 =$		27	$15 - 7 =$	
6	$38 - 7 =$		28	$25 - 7 =$	
7	$9 - 7 =$		29	$35 - 7 =$	
8	$19 - 7 =$		30	$75 - 7 =$	
9	$49 - 7 =$		31	$55 - 7 =$	
10	$10 - 7 =$		32	$16 - 7 =$	
11	$20 - 7 =$		33	$26 - 7 =$	
12	$60 - 7 =$		34	$36 - 7 =$	
13	$11 - 7 =$		35	$86 - 7 =$	
14	$21 - 7 =$		36	$66 - 7 =$	
15	$71 - 7 =$		37	$90 - 7 =$	
16	$12 - 7 =$		38	$53 - 7 =$	
17	$22 - 7 =$		39	$42 - 7 =$	
18	$82 - 7 =$		40	$71 - 7 =$	
19	$13 - 7 =$		41	$74 - 7 =$	
20	$23 - 7 =$		42	$56 - 7 =$	
21	$83 - 7 =$		43	$95 - 7 =$	
22	$14 - 7 =$		44	$92 - 7 =$	

**B** Improvement \_\_\_\_\_ # Correct \_\_\_\_\_

Subtract.					
1	$7 - 7 =$		23	$24 - 7 =$	
2	$17 - 7 =$		24	$34 - 7 =$	
3	$27 - 7 =$		25	$54 - 7 =$	
4	$8 - 7 =$		26	$74 - 7 =$	
5	$18 - 7 =$		27	$15 - 7 =$	
6	$68 - 7 =$		28	$25 - 7 =$	
7	$9 - 7 =$		29	$35 - 7 =$	
8	$19 - 7 =$		30	$65 - 7 =$	
9	$79 - 7 =$		31	$45 - 7 =$	
10	$10 - 7 =$		32	$16 - 7 =$	
11	$20 - 7 =$		33	$26 - 7 =$	
12	$90 - 7 =$		34	$36 - 7 =$	
13	$11 - 7 =$		35	$76 - 7 =$	
14	$21 - 7 =$		36	$56 - 7 =$	
15	$91 - 7 =$		37	$70 - 7 =$	
16	$12 - 7 =$		38	$63 - 7 =$	
17	$22 - 7 =$		39	$52 - 7 =$	
18	$42 - 7 =$		40	$81 - 7 =$	
19	$13 - 7 =$		41	$74 - 7 =$	
20	$23 - 7 =$		42	$66 - 7 =$	
21	$43 - 7 =$		43	$85 - 7 =$	
22	$14 - 7 =$		44	$52 - 7 =$	

Name \_\_\_\_\_

Date \_\_\_\_\_

1. Use the tape diagram to model equivalent fractions. Fill in the blanks and answer the following questions.



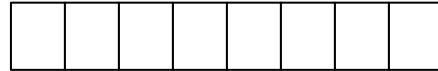
4 sixths is equal to \_\_\_\_\_ thirds.

$$\frac{4}{6} = \frac{\square}{3}$$

The whole stays the same.

What happened to the size of the equal parts when there were less equal parts?

What happened to the number of equal parts when the equal parts became larger?



1 half is equal to \_\_\_\_\_ eighths.

$$\frac{1}{2} = \frac{\square}{8}$$

The whole stays the same.

What happened to the size of the equal parts when there were more equal parts?

What happened to the number of equal parts when the equal parts became smaller?

2. 6 friends want to share three chocolate bars that are all the same size, represented by the 3 strips below. When the bars are unwrapped, the girls notice that the first chocolate bar is cut into 2 equal parts, the second is cut into 4 equal parts, and the third is cut into 6 equal parts. How can the 6 friends share the chocolate bars equally, without breaking any of the pieces?





3. When the whole is the same, why does it take 6 copies of  $\frac{1}{8}$  to show 3 copies of  $\frac{1}{4}$ ? Draw a model to support your answer.
4. When the whole is the same, how many sixths does it take to make  $\frac{1}{3}$ ? Draw a model to support your answer.
5. You have a magic wand that doubles the number of equal parts but keeps the whole the same size. Use your magic wand. In the space below draw to show what happens to a rectangle that is partitioned in fourths after you tap it with your wand. Use words and numbers to explain what happened.



Name \_\_\_\_\_

Date \_\_\_\_\_

1. Solve.

2 thirds is equal to \_\_\_\_\_ twelfths.

$$\frac{2}{3} = \frac{\quad}{12}$$

2. Draw and label two models that show fractions equivalent to those in Problem 1.

3. Use words to explain why the two fractions in Problem 1 are equal.

Name \_\_\_\_\_

Date \_\_\_\_\_

1. Use the tape diagram to model equivalent fractions. Fill in the blanks and answer the following questions.



2 tenths is equal to \_\_\_\_\_ fifths.

$$\frac{2}{10} = \frac{\quad}{5}$$

The whole stays the same.

What happened to the size of the equal parts when there were less equal parts?



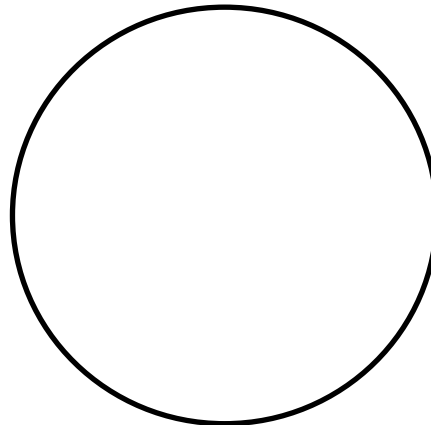
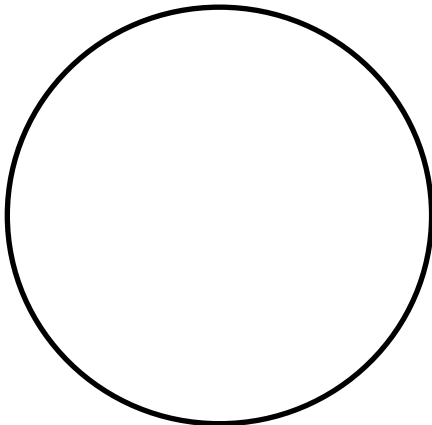
1 third is equal to \_\_\_\_\_ ninths.

$$\frac{1}{3} = \frac{\quad}{9}$$

The whole stays the same.

What happened to the size of the equal parts when there were more equal parts?

2. 8 students want to share 2 pizzas that are the same size, represented by the 2 circles below. They notice that the first pizza is cut into 4 equal slices, and the second is cut into 8 equal slices. How can the 8 students share the pizzas equally, without breaking any of the pieces?



3. When the whole is the same, why does it take 4 copies of 1 tenth to show 2 copies of 1 fifth? Draw a model to support your answer.
4. When the whole is the same, how many eighths does it take to make 1 fourth? Draw a model to support your answer.
5. Mr. Pham cuts a cake into 8 equal slices. Then he cuts every slice in half. How many of the small slices does he have? Use words and numbers to explain your answer.