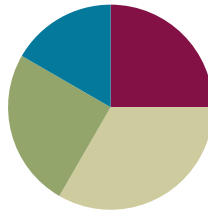# Lesson 13

Objective: Identify and use arithmetic patterns to multiply.

Related Topics: [More Lesson Plans for the Common Core Math](#)

Suggested Lesson Structure

■ Fluency Practice	(15 minutes)
■ Concept Development	(20 minutes)
■ Application Problem	(15 minutes)
■ Student Debrief	(10 minutes)
Total Time	**(60 minutes)**

> **NOTES ON LESSON STRUCTURE:**
>
> The Problem Set follows immediately after the Application Problem in this lesson.
>
> The 10 minutes for the Problem Set are included in the time allotted for the Application Problem rather than the Concept Development.

Fluency Practice (15 minutes)

- Sprint: Divide by 8 **3.OA.7**   (8 minutes)
- Group Counting **3.OA.1**   (4 minutes)
- Decompose Multiples of 9 **3.OA.5**   (3 minutes)

Sprint: Divide by 8 (8 minutes)

Materials: (S) Divide by 8 Sprint

Note: This Sprint reviews Lessons 10 and 11, focusing on the relationship between multiplying and dividing using units of 8.

Group Counting (4 minutes)

Note: Group counting reviews interpreting multiplication as repeated addition. Counting by sixes, sevens, and eights reviews multiplication taught earlier in the module. Group counting nines prepares students for multiplication in this lesson. Direct students to count forward and backward, occasionally changing the direction of the count:

- Sixes to 60
- Sevens to 70
- Eights to 80
- Nines to 90

> **A NOTE ON STANDARDS ALIGNMENT:**
>
> Some problems in the Sprint extend beyond Grade 3 multiplication and division standards, because students who have mastered their times tables are likely to otherwise go unchallenged during this activity. By extending to products above 90 and quotients above 10, strong students are provided the stimulus to stretch their conceptual understanding, which will likely keep them engaged and invigorated to improve on Sprint B.

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Decompose Multiples of 9 (3 minutes)

Materials: (S) Personal white boards

Note: This activity prepares students to use the distributive property using units of 9.

T: (Project a number bond with a whole of 45 and a part of 18.) On your boards, complete the missing part in the number bond.

S: (Write 27.)

Continue with the following possible sequence: whole of 90 and 27 as a part, whole of 54 and 36 as a part, whole of 72 and 27 as a part, whole of 63 and 18 as a part.

Concept Development (20 minutes)

Materials: (S) Personal white boards

Part 1: Identify patterns in multiples of 9.

T: During the fluency, we group counted nines to say the **multiples** of 9. When we skip-count by nines, what are we adding each time?

S: 9!

T: Adding nines can be tricky. What's a simplifying strategy for adding 9?

S: I can break apart 9 to make the total a ten, then add what's left of the 9 to it. → I can add 10, then subtract 1.

T: (Lead students through applying the add 10, subtract 1 strategy in Problem 2 on the Problem Set. Model the first example. Students can then work in pairs to find the rest. Allow time for students to finish their work.)

T: Compare the digits in the ones and tens places of the multiples. What pattern do you notice?

S: The digit in the tens place increases by 1. → The digit in the ones place decreases by 1.

MP.7

T: Now, with your partner, analyze the sum of the digits for each multiple of 9. What pattern do you notice?

S: Every multiple has a sum of the digits equal to 9!

T: How can knowing this help you with nines facts?

S: To check my answer I can add up the digits. If the sum isn't equal to 9, I made a mistake.

⑨
$9 + 10 = 19$
⑱ 9 is 1 less than 10 so:
$9 + 9 = 18$

$18 + 10 = 28$
㉗ 9 is 1 less than 10 so:
$18 + 9 = 27$

$27 + 10 = 37$
㊱ 9 is 1 less than 10 so:
$27 + 9 = 36$

$36 + 10 = 46$
㊺ 9 is 1 less than 10 so:
$36 + 9 = 45$

$45 + 10 = 55$
㊾ 9 is 1 less than 10 so:
$45 + 9 = 54$

Etc.

> **NOTES ON MULTIPLE MEANS OF REPRESENTATION:**
>
> Simplify and clarify "sum of the digits" for English language learners and others. Distinguish *some* from *sum*. Perhaps express the request in another manner, such as, "Look at the product. (Point.) Add the number in the ones place to the number in the tens place. What's the sum? Now, look at the next product… "

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Part 2: Apply strategies to solve nines facts.

Have students write and solve all facts from 1 × 9 to 10 × 9 in a column on their boards.

$1 \times 9 = 9$
$2 \times 9 = 18$
$3 \times 9 = 27$
$4 \times 9 = 36$
$5 \times 9 = 45$
$6 \times 9 = 54$
$7 \times 9 = 63$
$8 \times 9 = 72$
$9 \times 9 = 81$
$10 \times 9 = 90$

> T: Let's examine 1 × 9 = 9. Here, what is 9 multiplied by?
>
> S: 9 is multiplied by 1.
>
> T: What number is in the tens place of the product for 1 × 9?
>
> S: Zero.
>
> T: How is the number in the tens place related to 1?
>
> S: It is 1 less. → Zero is one less than 1.
>
> T: Say the product of 2 × 9 at my signal. (Signal.)
>
> S: 18.
>
> T: Which digit is in the tens place of the product?
>
> S: 1.
>
> T: How is the digit in the tens place related to the 2?
>
> S: It's one less again. → 1 is one less than 2!

Repeat the process with 3 × 9 and 4 × 9.

> T: What pattern do you notice with the digit in the tens place for each of those products?
>
> S: The number in the tens place is 1 less than the number of groups.
>
> T: With your partner, see if that pattern fits for the rest of the nines facts to ten.
>
> S: It does! The pattern keeps going!
>
> T: Let's see if we can find a pattern involving the ones place. We know that 2 × 9 equals 18. The 2 and 8 are related in some way. We also know that 3 × 9 equals 27. The 3 and 7 are related in the *same* way. Discuss with your partner, how are they related?
>
> S: 2 + 8 = 10 and 3 + 7 = 10! → 10 − 2 = 8 and 10 − 3 = 7!
>
> T: When you take the number of groups and subtract it from 10, what do you get?
>
> S: The ones place in the product!
>
> T: With your partner, see if that pattern fits for the rest of the nines facts on your board. (Allow students time to finish their work.)
>
> T: Did the pattern work for every fact, 1 × 9 through 10 × 9?
>
> S: Yes!
>
> T: Let's try 11 × 9. What is the product?
>
> S: 99!
>
> T: What is the number of groups?
>
> S: 11!
>
> T: Talk to your partner: Does the pattern work for 11 × 9? Why or why not?
>
> S: No, the pattern doesn't make sense. You can't have 10 in the tens place, and we don't know how to solve 10 − 11 to find what digit is in the ones place.
>
> T: The pattern can give you the answer to any nines fact from 1 × 9 to 10 × 9, but it doesn't work for nines facts bigger than 10 × 9.

Application Problem (15 minutes)

Michaela and Gilda read the same book. It takes Michaela about 8 minutes to read a chapter, and Gilda about 10 minutes. There are 9 chapters in the book. How many fewer minutes does Michaela spend reading than Gilda?

Michaela $8 \times 9 = 72$ minutes

Gilda $10 \times 9 = 90$ minutes

$90 - 72 = 18$ minutes

Michaela spends 18 fewer minutes reading.

Note: This problem comes after the Concept Development so that students have the opportunity to apply some of the strategies they learned in the context of problem solving. Encourage them to check their answers to the nines facts using new learning.

Problem Set (10 minutes)

Students should do their personal best to complete the Problem Set within the allotted 10 minutes. For some classes, it may be appropriate to modify the assignment by specifying which problems they work on first. Some problems do not specify a method for solving. Students solve these problems using the RDW approach used for Application Problems.

> **NOTES ON MULTIPLE MEANS OF REPRESENTATION:**
>
> You may adjust the Problem Set to assist learners with perceptual disabilities. Provide a table for students to record the nine skip-count, and/or highlight the tens or ones.
> As students solve Problem 2, it may be helpful to write the nines fact for each given product, e.g. in Problem 2(f), have students write 5 × 9 above 45.

Student Debrief (10 minutes)

Lesson Objective: Identify and use arithmetic patterns to multiply.

The Student Debrief is intended to invite reflection and active processing of the total lesson experience.

Invite students to review their solutions for the Problem Set. They should check work by comparing answers with a partner before going over answers as a class. Look for misconceptions or misunderstandings that can be addressed in the Debrief. Guide students in a conversation to debrief the Problem Set and process the lesson.

You may choose to use any combination of the questions below to lead the discussion.

- What patterns did you use to solve Problem 1?
- The add 10, subtract 1 strategy can be used to quickly find **multiples** of 9. How could you change it to quickly find multiples of 8?
- How is the add 10, subtract 1 strategy related to the 9 = 10 – 1 break apart and distribute strategy we learned recently?
- In Problem 3(d) how did you figure out where Kent's strategy stops working? Why doesn't this strategy

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work past 10 × 9?

- How can the number of groups in a nines fact help you find the product?
- How did Group Counting during the Fluency help get us ready for today's lesson?

Exit Ticket (3 minutes)

After the Student Debrief, instruct students to complete the Exit Ticket. A review of their work will help you assess the students' understanding of the concepts that were presented in the lesson today and plan more effectively for future lessons. You may read the questions aloud to the students.

A

Correct _____

Multiply or divide.

1	2 x 8 =		23	__ x 8 = 80	
2	3 x 8 =		24	__ x 8 = 32	
3	4 x 8 =		25	__ x 8 = 24	
4	5 x 8 =		26	80 ÷ 8 =	
5	1 x 8 =		27	40 ÷ 8 =	
6	16 ÷ 8 =		28	8 ÷ 1 =	
7	24 ÷ 8 =		29	16 ÷ 8 =	
8	40 ÷ 8 =		30	24 ÷ 8 =	
9	8 ÷ 1 =		31	__ x 8 = 48	
10	32 ÷ 8 =		32	__ x 8 = 56	
11	6 x 8 =		33	__ x 8 = 72	
12	7 x 8 =		34	__ x 8 = 64	
13	8 x 8 =		35	56 ÷ 8 =	
14	9 x 8 =		36	72 ÷ 8 =	
15	10 x 8 =		37	48 ÷ 8 =	
16	64 ÷ 8 =		38	64 ÷ 8 =	
17	56 ÷ 8 =		39	11 x 8 =	
18	72 ÷ 8 =		40	88 ÷ 8 =	
19	48 ÷ 8 =		41	12 x 8 =	
20	80 ÷ 8 =		42	96 ÷ 8 =	
21	__ x 8 = 40		43	14 x 8 =	
22	__ x 8 = 16		44	112 ÷ 8 =	

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B

Improvement _____   # Correct _____

Multiply or divide.

1	1 x 8 =		23	__ x 8 = 48	
2	2 x 8 =		24	__ x 8 = 80	
3	3 x 8 =		25	__ x 8 = 24	
4	4 x 8 =		26	16 ÷ 8 =	
5	5 x 8 =		27	8 ÷ 1 =	
6	24 ÷ 8 =		28	80 ÷ 8 =	
7	16 ÷ 8 =		29	40 ÷ 8 =	
8	32 ÷ 8 =		30	24 ÷ 8 =	
9	8 ÷ 1 =		31	__ x 8 = 64	
10	40 ÷ 8 =		32	__ x 8 = 32	
11	10 x 8 =		33	__ x 8 = 72	
12	6 x 8 =		34	__ x 8 = 56	
13	7 x 8 =		35	64 ÷ 8 =	
14	8 x 8 =		36	72 ÷8 =	
15	9 x 8 =		37	48 ÷ 8 =	
16	56 ÷ 8 =		38	56 ÷ 8 =	
17	48 ÷ 8 =		39	11 x 8 =	
18	64 ÷ 8 =		40	88 ÷ 8 =	
19	80 ÷ 8 =		41	12 x 8 =	
20	72 ÷8 =		42	96 ÷ 8 =	
21	__ x 8 = 16		43	13 x 8 =	
22	__ x 8 = 40		44	104 ÷ 8 =	

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Name _____   Date _____

1. a.   Skip-count by nine.

   ___**9**___, _____, _____, ___**36**___, _____, _____, _____, ___**72**___, _____, _____

   b.   Look at the **tens** place in the count-by. What is the pattern?

   c.   Look at the **ones** place in the count-by. What is the pattern?

2. Complete to make true statements.

   a.   10 more than 0 is ____**10**__,

   1 less is ___**9**___.

   1 × 9 = ___**9**___

   b.   10 more than 9 is ___**19**___,

   1 less is ___**18**___.

   2 × 9 = _____

   c.   10 more than 18 is _____,

   1 less is _____.

   3 × 9 = _____

   d.   10 more than 27 is _____,

   1 less is _____.

   4 × 9 = _____

   e.   10 more than 36 is _____,

   1 less is _____.

   5 × 9 = _____

   f.   10 more than 45 is _____,

   1 less is _____.

   6 × 9 = _____

   g.   10 more than 54 is _____,

   1 less is _____.

   7 × 9 = _____

   h.   10 more than 63 is _____,

   1 less is _____.

   8 × 9 = _____

   i.   10 more than 72 is _____,

   1 less is _____.

   9 × 9 = _____

   j.   10 more than 81 is _____,

   1 less is _____.

   10 × 9 = _____

3. a. Analyze the equations in Problem 2. What is the pattern?

b. Use the pattern to find the next 4 facts. Show your work.

$11 \times 9 =$ $\qquad$ $12 \times 9 =$ $\qquad$ $13 \times 9 =$ $\qquad$ $14 \times 9 =$

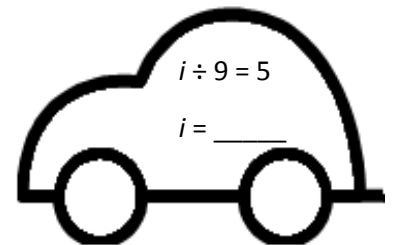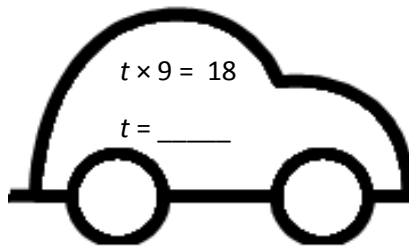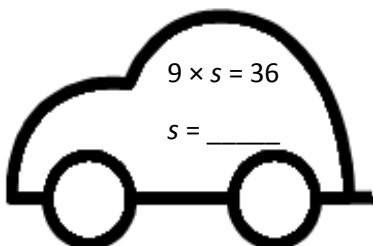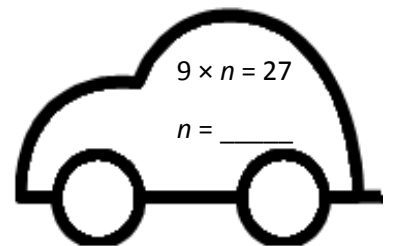c. Kent notices another pattern in Problem 2. His work is shown below. He sees that:
   ▪ the tens digit in the product is 1 less than the number of groups
   ▪ the ones digit in the product is 10 minus the number of groups

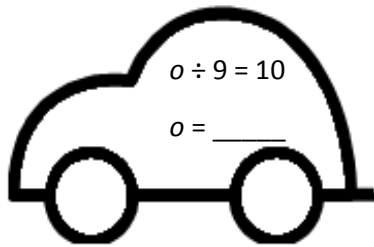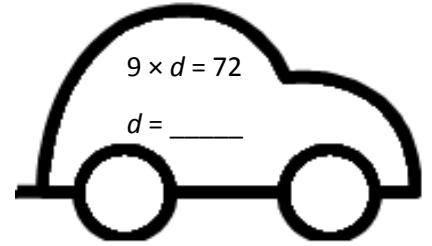|	tens digit	ones digit
$2 \times 9 = \underline{\mathbf{18}}$ →	$\underline{\mathbf{1}} = 2 - 1$	$\underline{\mathbf{8}} = 10 - 2$
$3 \times 9 = \underline{\mathbf{27}}$ →	$\underline{\mathbf{2}} = 3 - 1$	$\underline{\mathbf{7}} = 10 - 3$
$4 \times 9 = \underline{\mathbf{36}}$ →	$\underline{\mathbf{3}} = 4 - 1$	$\underline{\mathbf{6}} = 10 - 4$
$5 \times 9 = \underline{\mathbf{45}}$ →	$\underline{\mathbf{4}} = 5 - 1$	$\underline{\mathbf{5}} = 10 - 5$

Use Kent's strategy to solve 6×9 and 7×9.

d. Show an example of when Kent's pattern doesn't work.

4. Each number sentence contains a letter representing the unknown. Find the value of each unknown. Then write the letters that match the answers to solve the riddle.

$a \times 9 = 54$

$a = $ _____

$81 \div 9 = g$

$g = $ _____

$9 \times d = 72$

$d = $ _____

$o \div 9 = 10$

$o = $ _____

$e \times 9 = 63$

$e = $ _____

$9 \times n = 27$

$n = $ _____

$9 \times s = 36$

$s = $ _____

$t \times 9 = 18$

$t = $ _____

$i \div 9 = 5$

$i = $ _____
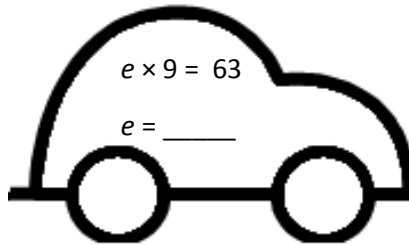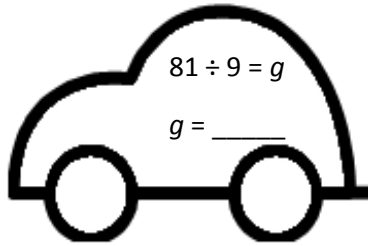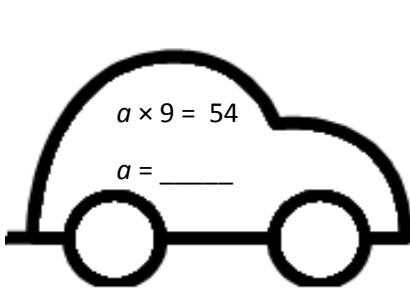
How do you make one vanish?

___ ___ ___ ___ ' ___ ' ___ ___ ___ ___ ___ ' ___ ___ ___ ___ ___ !
6   8   8   6   9   6   3   8   45   2   4   9   90   3   7

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Name _____   Date _____

1. $6 \times 9 = 54$ $8 \times 9 = 72$

   What is 10 more than 54? _____   What is 10 more than 72? _____

   What is 1 less? _____   What is 1 less? _____

   $7 \times 9 =$ _____ $9 \times 9 =$ _____

2. Explain the pattern used in Problem 1.

Name _____   Date _____

1. a. Skip-count by nines down from 90.

   __**90**__, _____, __**72**__, _____, _____, _____, __**36**__, _____, _____, _____

   b. Look at the **tens** place in the count-by. What is the pattern?

   c. Look at the ones place in the count-by. What is the pattern?

2. Each number sentence contains a letter representing the unknown. Find the value of each unknown.

   $a \times 9 = 18$

   $a =$ _____

   $m \div 9 = 3$

   $m =$ _____

   $e \times 9 = 45$

   $e =$ _____

   $f \div 9 = 4$

   $f =$ _____

   $9 \times d = 81$

   $d =$ _____

   $w \div 9 = 6$

   $w =$ _____

   $9 \times s = 90$

   $s =$ _____

   $k \div 9 = 8$

   $k =$ _____

3. Solve.

a. What is 10 more than 0? _____

What is 1 less? _____

1 × 9 = _____

b. What is 10 more than 9? _____

What is 1 less? _____

2 × 9 = _____

c. What is 10 more than 18? _____

What is 1 less? _____

3 × 9 = _____

d. 10 more than 27? _____

What is 1 less? _____

4 × 9 = _____

e. What is 10 more than 36? _____

What is 1 less? _____

5 × 9 = _____

f. What is 10 more than 45? _____

What is 1 less? _____

6 × 9 = _____

g. What is 10 more than 54? _____

What is 1 less? _____

7 × 9 = _____

h. What is 10 more than 63? _____

What is 1 less? _____

8 × 9 = _____

i. What is 10 more than 72? _____

What is 1 less? _____

9 × 9 = _____

j. What is 10 more than 81? _____

What is 1 less? _____

10 × 9 = _____

4. Explain the pattern in Problem 2 and use the pattern to find the next 3 facts.

11 × 9 = _____

12 × 9 = _____

13 × 9 = _____

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