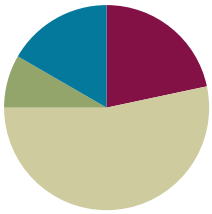
Lesson 7

Objective: Demonstrate the commutativity of multiplication and practice related facts by skip-counting objects in array models.

Related Topics: More Lesson Plans for the Common Core Math

Suggested Lesson Structure

Fluency Practice	(13 minutes)
Application Problem	(5 minutes)
Concept Development	(32 minutes)
Student Debrief	(10 minutes)
Total Time	**(60 minutes)**



Fluency Practice (13 minutes)

- Group Counting **3.OA.1** (4 minutes)
- Divide Equal Groups **3.OA.2** (5 minutes)
- Multiply with Twos **3.OA.7** (4 minutes)

Group Counting (4 minutes)

Note: Group counting reviews interpreting multiplication as repeated addition. Counting by twos and threes in this activity reviews work with those factors from Topic C.

T: Let's count by twos. (Direct students to count forward and backward to 20, emphasizing the 8 to 10, 10 to 12, and 18 to 20 transitions.)

T: Let's count by threes. (Direct students to count forward and backward to 30, periodically changing directions. Emphasize the 9 to 12, 18 to 21, and 27 to 30 transitions.)

Divide Equal Groups (5 minutes)

Materials: (S) Personal white boards

Note: Students directly relate repeated addition to division. They interpret the unknown in division. This activity reviews Lesson 6.

T: (Project a picture array with 2 groups of 4 circled.) Say the total as a repeated addition sentence.

S: 4 + 4 = 8.

T: Write a division sentence for 8 divided into 2 equal groups.

S: (Write 8 ÷ 2 = 4.)

T: Below that division sentence write a division sentence dividing 8 into 4 equal groups.
S: (Write 8 ÷ 4 = 2.)

Continue with possible sequence: 5 groups of 3, 3 groups of 4, and 6 groups of 2.

Multiply with Twos (4 minutes)

Materials: (S) Personal white boards, Twos Array Template, blank paper

Note: Students unit count objects in an array and write multiplication sentences that match the count-by in anticipation of this lesson's objective.

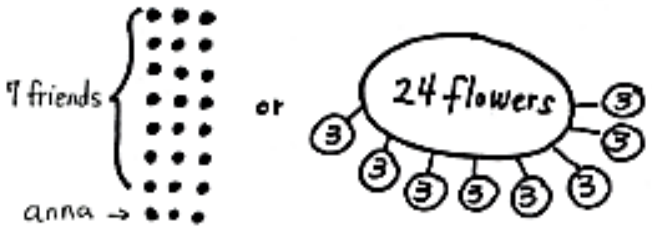
T: Slip your Twos Array Template into your personal white board.
T: Turn your board so that it's vertical. Use your blank paper to cover all but the first row of dots.
T: How many twos show?
S: 1 two.
T: Say the multiplication sentence to represent the shown array and solve.
S: 1 × 2 = 2.
T: Uncover another row.

Continue this sequence having students uncover twos for 2 × 2, 3 × 2, 10 × 2, 5 × 2, 6 × 2, 7 × 2, 9 × 2, and 8 × 2.

Application Problem (5 minutes)

Anna picks 24 flowers. She makes equal bundles of flowers and gives 1 bundle to each of her 7 friends. She keeps a bundle for herself too. How many flowers does Anna put in each bundle?

Note: This problem reviews equal groups division from Lesson 5 where the unknown represents the size of the group. The problem's complexity is in understanding that the flowers are divided equally into 8 bundles, not 7, since they need to count Anna. Students may choose to solve by drawing a division array learned in Lesson 6 or a number bond learned in Lesson 3.



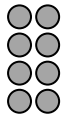
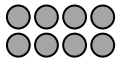
24 ÷ 8 = 3
Anna puts 3 flowers in each bundle.

Concept Development (32 minutes)

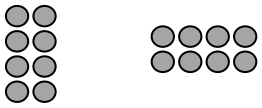
Materials: (S) Personal white boards

Problem 1: Rotate arrays 90 degrees.

Final Student Personal White Board

2, 4, 6, 8	4, 8
	
4 × 2 =	2 × 4 =

- T: Turn your personal white board so it's horizontal. Draw a line down the middle to make 2 sides. On the left side skip-count by two 4 times and write each number.
- S: (Write 2, 4, 6, 8.)
- T: On the right side skip-count by four 2 times.
- S: (Write 4, 8.)
- T: How are the count-bys related?
- S: The first one is 4 twos and the second one is 2 fours.
- T: Under each count-by draw an array to match it.
- S: (Draw arrays shown below.)



- T: What do you notice about the arrays? Do they both have 4 groups of 2?
- S: (Discuss.)
- T: Do they both have 2 groups of 4?
- S: Yes. The one on the right has 2 rows of 4. If you turn it sideways then the one on the left does too. Or you can just see that it has 2 vertical rows of 4.
- T: It's the same array turned different ways. We have a special name for rows when they are vertical. We call them **columns**.

NOTES ON MULTIPLE MEANS OF REPRESENTATION:

If necessary students may keep the Twos Array Template from the fluency activity *Multiply with Two* alongside their workspace to help them draw arrays from skip-counting.

Prompt students to write and solve multiplication sentences to show the total objects in each array. Continue with the following possible examples:

2 × 5 array

7 × 2 array

As you circulate, guide them to notice that factors switch places and help them to relate the change to the rotated array. For example, write 4 × 2 = 2 × 4 and ask students to discuss how they know it is true.

- T: Depending on how we look at an array, columns *or* rows can be the number of groups. Discuss with your partner how you know that's true.
- S: (Discuss.)

Problem 2: Interpreting rows and columns in rotated arrays.

T: Turn your board so it's vertical. Draw an array that shows 8 equal groups of two. How many rows of 2 did you draw?
S: 8 rows.
T: How many columns of 8 did you draw?
S: 2 columns.
T: Write a multiplication sentence to match the array. Don't solve it yet.
S: (Write 8 × 2 = ____.)
T: **Rotate** your board so that it's horizontal. How many rows of 8 do you have now?
S: 2 rows.
T: How many columns of 2?
MP.7 S: 8 columns.
T: Write a multiplication sentence to match the array. Don't solve it yet.
S: (Write 2 × 8 = ____.)
T: Explain to your partner using the words *columns* and *rows* why your multiplication sentence changed.
S: When the array turned the columns and rows switched. → Columns became rows and rows became columns. They both represent equal groups. It depends on how you look at the array.
T: Will 8 × 2 and 2 × 8 have the same total?
S: Yes!
T: How do you know?
S: They have the same array. → 2 groups of 8 and 8 groups of 2 are the same.

(Prompt students to skip-count to find the totals of the array in both positions.)

Work through the following examples to build vocabulary and understanding of commutativity: 6 × 2, 2 × 9

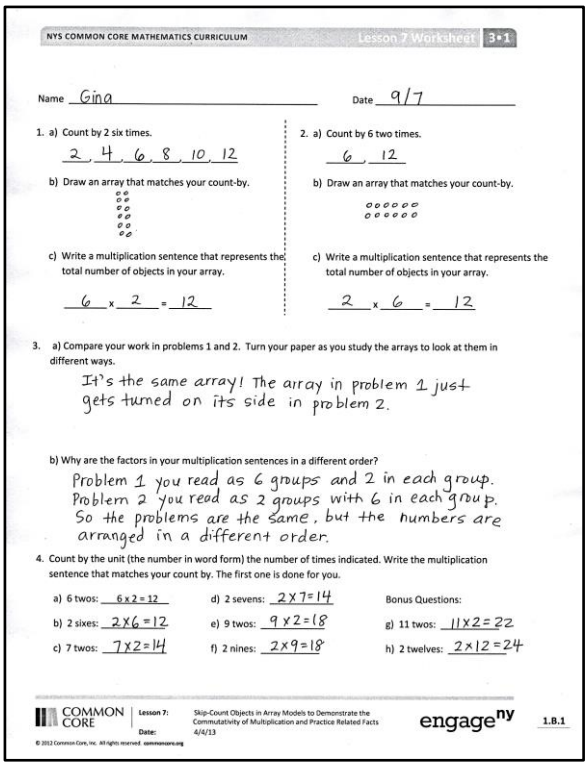
T: When we multiply, changing the order of the factors doesn't change the total. We say the factors are **commutative**. That means they can switch around. Tell your partner what commutative means.
S: It means numbers can switch around. → The factors change places in a multiplication sentence, but the total doesn't change. → Addition works the same way.
T: What we've explored today is called the **commutative property**.

NOTES ON MULTIPLE MEANS OF REPRESENTATION:

Students need not master the words *commutative* or *commutative property.* However, they will need to be familiar with the vocabulary moving forward in this module.

Problem Set (10 minutes)

Students should do their personal best to complete the Problem Set within the allotted 10 minutes. For some classes, it may be appropriate to modify the assignment by specifying which problems they work on first. Some problems do not specify a method for solving. Students solve these problems using the RDW approach used for Application Problems.



NYS COMMON CORE MATHEMATICS CURRICULUM Lesson 7 Worksheet 3•1

Name Gina Date 9/7

1. a) Count by 2 six times. 2, 4, 6, 8, 10, 12
b) Draw an array that matches your count-by.
c) Write a multiplication sentence that represents the total number of objects in your array. 6 x 2 = 12

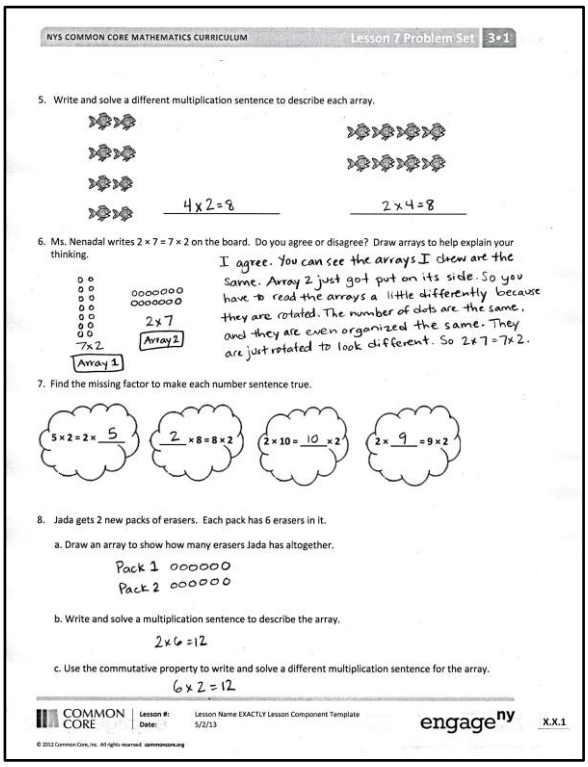
2. a) Count by 6 two times. 6, 12
b) Draw an array that matches your count-by.
c) Write a multiplication sentence that represents the total number of objects in your array. 2 x 6 = 12

3. a) Compare your work in problems 1 and 2. Turn your paper as you study the arrays to look at them in different ways.
It's the same array! The array in problem 1 just gets turned on its side in problem 2.

b) Why are the factors in your multiplication sentences in a different order?
Problem 1 you read as 6 groups and 2 in each group. Problem 2 you read as 2 groups with 6 in each group. So the problems are the same, but the numbers are arranged in a different order.

4. Count by the unit (the number in word form) the number of times indicated. Write the multiplication sentence that matches your count by. The first one is done for you.
a) 6 twos: 6x2=12
b) 2 sixes: 2x6=12
c) 7 twos: 7x2=14
d) 2 sevens: 2x7=14
e) 9 twos: 9x2=18
f) 2 nines: 2x9=18
Bonus Questions:
g) 11 twos: 11x2=22
h) 2 twelves: 2x12=24

COMMON CORE Lesson 7: Skip-Count Objects in Array Models to Demonstrate the Commutativity of Multiplication and Practice Related Facts Date: 4/4/13 engage^ny 1.B.1



NYS COMMON CORE MATHEMATICS CURRICULUM Lesson 7 Problem Set 3•1

5. Write and solve a different multiplication sentence to describe each array.
4x2=8
2x4=8

6. Ms. Nenadal writes 2 × 7 = 7 × 2 on the board. Do you agree or disagree? Draw arrays to help explain your thinking.
I agree. You can see the arrays I drew are the same. Array 2 just got put on its side. So you have to read the arrays a little differently because they are rotated. The number of dots are the same, and they are even organized the same. They are just rotated to look different. So 2x7=7x2.
7x2 Array 1
2x7 Array 2

7. Find the missing factor to make each number sentence true.
5 × 2 = 2 × 5
2 × 8 = 8 × 2
2 × 10 = 10 × 2
2 × 9 = 9 × 2

8. Jada gets 2 new packs of erasers. Each pack has 6 erasers in it.
a. Draw an array to show how many erasers Jada has altogether.
Pack 1 ooooooo
Pack 2 ooooooo
b. Write and solve a multiplication sentence to describe the array.
2x6=12
c. Use the commutative property to write and solve a different multiplication sentence for the array.
6x2=12

COMMON CORE Lesson #: Lesson Name EXACTLY Lesson Component Template Date: 5/2/13 engage^ny X.X.1

Student Debrief (10 minutes)

Lesson Objective: Demonstrate the commutativity of multiplication and practice related facts by skip-counting objects in array models.

The Student Debrief is intended to invite reflection and active processing of the total lesson experience. Invite students to review their solutions for the Problem Set. They should check work by comparing answers with a partner before going over answers as a class. Look for misconceptions or misunderstandings that can be addressed in the Debrief. Guide students in a conversation to debrief the Problem Set and process the lesson. You may choose to use any combination of the ideas below to lead the discussion.

- Discuss the usefulness of unit counting to solve multiplication facts.
- Build fluency by having students demonstrate unit counting to find the answer to the following facts without the help of an array. They can keep track of the count using fingers.
  - 3 twos, 2 threes
  - 4 twos, 2 fours
  - 2 eights, 8 twos
  - 2 tens, 10 twos
- Review the vocabulary **columns** in contrast with *rows.*
- Discuss the **commutativity** of multiplication and how it relates to equal groups, columns, rows and **rotating** arrays.

- Relate the **commutative property** of multiplication to the commutative property of addition to help students recognize it in their prior learning.

Exit Ticket (3 minutes)

After the Student Debrief, instruct students to complete the Exit Ticket. A review of their work will help you assess the students' understanding of the concepts that were presented in the lesson today and plan more effectively for future lessons. You may read the questions aloud to the students.

Name _____ Date _____

1. a. Count by twos 6 times.

_____, _____,_____,_____,_____,_____

b. Draw an array that matches your count-by.

c. Write a multiplication sentence that represents the total number of objects in your array.

_____ × _____ = _____

2. a. Count by sixes 2 times.

_____, _____

b. Draw an array that matches your count-by.

c. Write a multiplication sentence that represents the total number of objects in your array.

_____ × _____ = _____

3. a. Compare your work in Problems 1 and 2. Turn your paper as you study the arrays to look at them in different ways.

b. Why are the factors in your multiplication sentences in a different order?

4. Count by the unit (the number in word form) the number of times indicated. Write the multiplication sentence that matches your count by. The first one is done for you.

a. 6 twos: 6 × 2 = 12

b. 2 sixes: _____

c. 7 twos: _____

d. 2 sevens: _____

e. 9 twos: _____

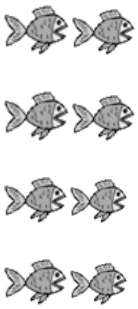
f. 2 nines: _____

Bonus Questions:

g. 11 twos: _____

h. 2 twelves: _____

5. Write and solve a different multiplication sentence to describe each array.



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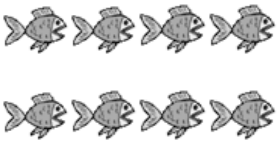
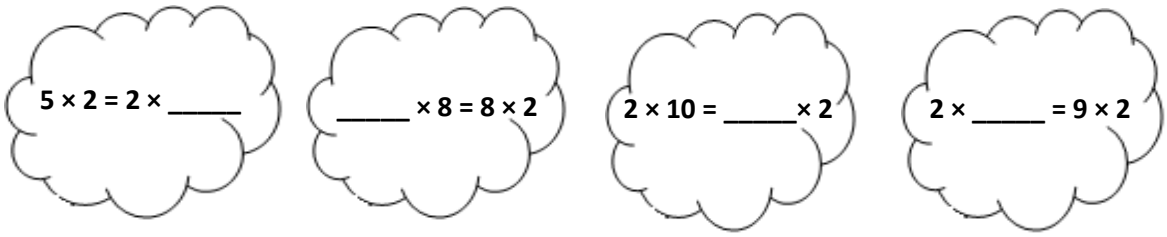


_____

6. Ms. Nenadal writes 2 × 7 = 7 × 2 on the board. Do you agree or disagree? Draw arrays to help explain your thinking.

7. Find the missing factor to make each number sentence true.



8. Jada gets 2 new packs of erasers. Each pack has 6 erasers in it.
   a. Draw an array to show how many erasers Jada has altogether.

   b. Write and solve a multiplication sentence to describe the array.

   c. Use the commutative property to write and solve a different multiplication sentence for the array.

Name _____ Date _____

$2 \times 5 = 5 \times 2$

Do you agree or disagree with the statement in the box? Draw arrays and use skip-counting to explain your thinking.

Name _____ Date _____

1. a. Count by twos 7 times.

_____, _____,_____,_____,_____,_____,_____

b. Draw an array that matches your count-by.

c. Write a multiplication sentence that represents the total number of objects in your array.

_____ × _____ = _____

2. a. Count by sevens 2 times.

_____, _____

b. Draw an array that matches your count-by.

c. Write a multiplication sentence that represents the total number of objects in your array.

_____ × _____ = _____

3. a. Compare your work in Problems 1 and 2. Turn your paper as you study the arrays to look at them in different ways.

b. Why are the factors in your multiplication sentences in a different order?

4. Count by the unit (the number in word form) the number of times indicated. Write the multiplication sentence that matches your count-by. The first one is done for you.

a. 2 twos: 2 × 2 = 4

b. 3 twos: _____

c. 2 threes: _____

d. 2 fours: _____

e. 4 twos: _____

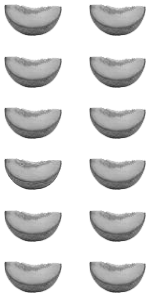
f. 5 twos: _____

g. 2 fives: _____

g. 6 twos: _____

h. 2 sixes: _____

5. Write and solve a different multiplication sentence to describe each array.



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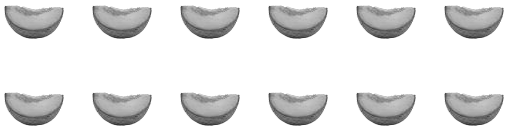

_____

6. Angel writes 2 × 8 = 8 × 2 in his notebook. Do you agree or disagree? Draw arrays to help explain your thinking.

7. Find the missing factor to make each number sentence true.

2 × 6 = 6 × _____	_____ × 2 = 2 × 7	9 × 2 = _____ × 9	2 × ____ = 10 × 2

8. Tamia buys 2 bags of candy. Each bag has 7 pieces of candy in it.
   a. Draw an array to show how many pieces of candy Tamia has altogether.

   b. Write and solve a multiplication sentence to describe the array.

   c. Use the commutative property to write and solve a different multiplication sentence for the array.