

Unit 7, Lesson 8: Combining Bases

Let's multiply expressions with different bases.

8.1: Same Exponent, Different Base

- 1. Evaluate $5^3 \cdot 2^3$
- 2. Evaluate 10^3

8.2: Exponent Product Rule

1. The table contains products of expressions with different bases and the same exponent. Complete the table to see how we can rewrite them. Use the "expanded" column to work out how to combine the factors into a new base.

expression	expanded	exponent
$5^3 \cdot 2^3$	$(5 \cdot 5 \cdot 5) \cdot (2 \cdot 2 \cdot 2) = (2 \cdot 5)(2 \cdot 5)(2 \cdot 5)$ = $10 \cdot 10 \cdot 10$	10 ³
$3^2 \cdot 7^2$		21 ²
$2^4 \cdot 3^4$		
		15 ³
		30 ⁴
$2^4 \cdot x^4$		
$a^n \cdot b^n$		
$7^4 \cdot 2^4 \cdot 5^4$		

2. What happens if neither the exponents nor the bases are the same? Can you write $2^3 \cdot 3^4$ with a single exponent? Explain or show your reasoning.

8.3: How Many Ways Can You Make 3,600?

Your teacher will give your group tools for creating a visual display to play a game. Divide the display into 3 columns, with these headers:

$$a^n \cdot a^m = a^{n+m}$$

$$\frac{a^n}{a^m} = a^{n-m}$$

$$a^n \cdot b^n = (a \cdot b)^n$$

How to play:

When the time starts, you and your group will write as many expressions as you can that equal a specific number using one of the exponent rules on your board. When the time is up, compare your expressions with another group to see how many points you earn.

- Your group gets 1 point for every *unique* expression you write that is equal to the number and follows the exponent rule you claimed.
- If an expression uses negative exponents, you get 2 points instead of just 1.
- You can challenge the other group's expression if you think it is not equal to the number or if it does not follow one of the three exponent rules.

Are you ready for more?

You have probably noticed that when you square an odd number, you get another odd number, and when you square an even number, you get another even number. Here is a way to expand the concept of odd and even for the number 3. Every integer is either divisible by 3, one MORE than a multiple of 3, or one LESS than a multiple of 3.

- 1. Examples of numbers that are one more than a multiple of 3 are 4, 7, and 25. Give three more examples.
- 2. Examples of numbers that are one less than a multiple of 3 are 2, 5, and 32. Give three more examples.
- 3. Do you think it's true that when you square a number that is a multiple of 3, your answer will still be a multiple of 3? How about for the other two categories? Try squaring some numbers to check your guesses.



Lesson 8 Summary

Before this lesson, we made rules for multiplying and dividing expressions with exponents that only work when the expressions have the *same* base. For example,

$$10^3 \cdot 10^2 = 10^5$$

or

$$2^6 \div 2^2 = 2^4$$

In this lesson, we studied how to combine expressions with the same exponent, but *different* bases. For example, we can write $2^3 \cdot 5^3$ as $2 \cdot 2 \cdot 2 \cdot 5 \cdot 5 \cdot 5$. Regrouping this as $(2 \cdot 5) \cdot (2 \cdot 5) \cdot (2 \cdot 5)$ shows that

$$2^3 \cdot 5^3 = (2 \cdot 5)^3 = 10^3$$

Notice that the 2 and 5 in the previous example could be replaced with different numbers or even variables. For example, if a and b are variables then $a^3 \cdot b^3 = (a \cdot b)^3$. More generally, for a positive number n,

$$a^n \cdot b^n = (a \cdot b)^n$$

because both sides have exactly n factors that are a and n factors that are b.

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1. Select **all** the true statements:

A.
$$2^8 \cdot 2^9 = 2^{17}$$

B.
$$8^2 \cdot 9^2 = 72^2$$

C.
$$8^2 \cdot 9^2 = 72^4$$

D.
$$2^8 \cdot 2^9 = 4^{17}$$

2. Find x, y, and z if $(3 \cdot 5)^4 \cdot (2 \cdot 3)^5 \cdot (2 \cdot 5)^7 = 2^x \cdot 3^y \cdot 5^z$.

3. Han found a way to compute complicated expressions more easily. Since $2 \cdot 5 = 10$, he looks for pairings of 2s and 5s that he knows equal 10. For example, $3 \cdot 2^4 \cdot 5^5 = 3 \cdot 2^4 \cdot 5^4 \cdot 5 = (3 \cdot 5) \cdot (2 \cdot 5)^4 = 15 \cdot 10^4 = 150,000$. Use Han's technique to compute the following:

a.
$$2^4 \cdot 5 \cdot (3 \cdot 5)^3$$

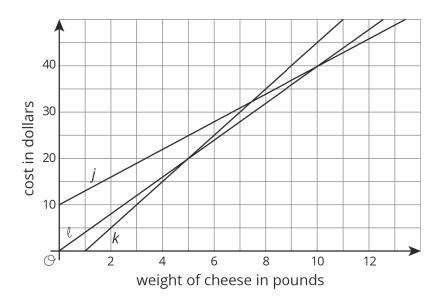
b.
$$\frac{2^3 \cdot 5^2 \cdot (2 \cdot 3)^2 \cdot (3 \cdot 5)^2}{3^2}$$

- 4. The cost of cheese at three stores is a function of the weight of the cheese. The cheese is not prepackaged, so a customer can buy any amount of cheese.
 - \circ Store A sells the cheese for a dollars per pound.
 - \circ Store B sells the same cheese for b dollars per pound and a customer has a coupon for \$5 off the total purchase at that store.
 - \circ Store C is an online store, selling the same cheese at c dollar per pound, but with a \$10 delivery



fee.

This graph shows the price functions for stores A, B, and C.



- a. Match Stores A, B, and C with Graphs j, k, and ℓ .
- b. How much does each store charge for the cheese per pound?
- c. How many pounds of cheese does the coupon for Store B pay for?
- d. Which store has the lowest price for a half a pound of cheese?
- e. If a customer wants to buy 5 pounds of cheese for a party, which store has the lowest price?
- f. How many pounds would a customer need to order to make Store C a good option?

(from Unit 5, Lesson 8)