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Unit 5, Lesson 9: Linear Models

Let’s model situations with linear functions.

9.1: Candlelight

m.openup.org/1/8-5-9-1

A candle is burning. It starts out 12 inches long. After 1 hour, it is 10 inches long. After 3 hours, it is 5.5 inches long.

1. When do you think the candle will burn out completely?
2. Is the height of the candle a function of time? If yes, is it a linear function? Explain your thinking.

9.2: Shadows

m.openup.org/1/8-5-9-2

When the sun was directly overhead, the stick had no shadow. After 20 minutes, the shadow was 10.5 cm long. After 60 minutes, it was 26 cm long.

1. Based on this information, estimate how long it will be after 95 minutes.
2. After 95 minutes, the shadow measured 38.5 cm. How does this compare to your estimate?
3. Is the length of the shadow a function of time? If so, is it linear? Explain your reasoning.

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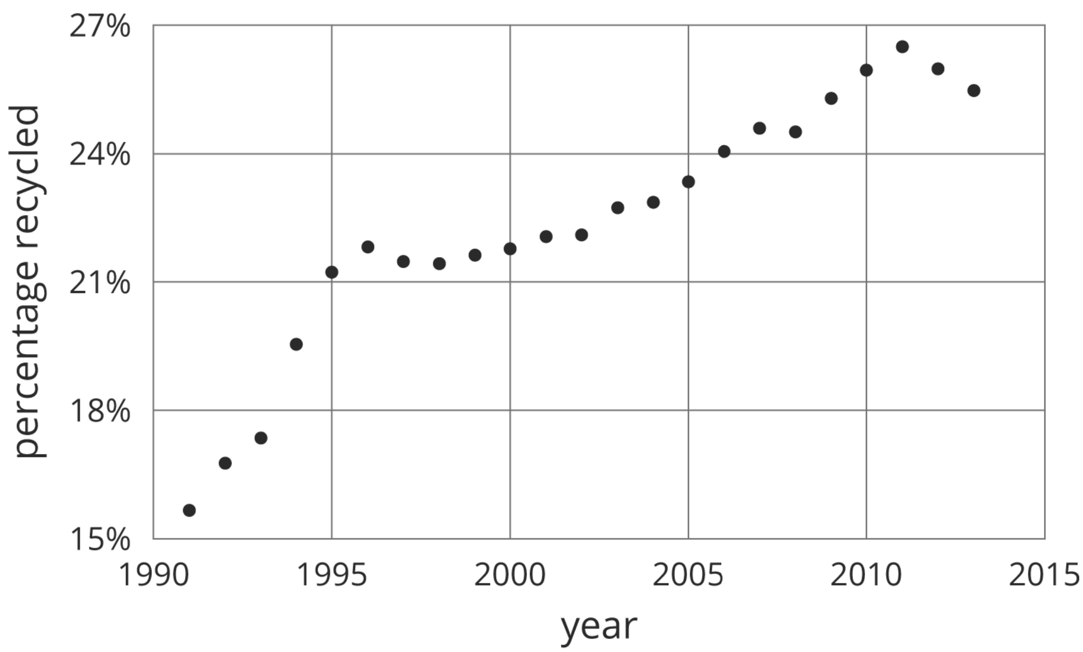
9.3: Recycling

In an earlier lesson, we saw this graph that shows the percentage of all garbage in the U.S. that was recycled between 1991 and 2013.



1. Sketch a linear function that models the change in the percentage of garbage that was recycled between 1991 and 1995. For which years is the model good at predicting the percentage of garbage that is produced? For which years is it not as good?

2. Pick another time period to model with a sketch of a linear function. For which years is the model good at making predictions? For which years is it not very good?

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Lesson 9 Summary

Water has different boiling points at different elevations. At 0 m above sea level, the boiling point is 100° C. At 2,500 m above sea level, the boiling point is 91.3° C. If we assume the boiling point of water is a linear function of elevation, we can use these two data points to calculate the slope of the line:

$$m = \frac{91.3 - 100}{2{,}500 - 0} = \frac{-8.7}{2{,}500}$$

This slope means that for each increase of 2,500 m, the boiling point of water decreases by 8.7° C. Next, we already know the y-intercept is 100° C from the first point, so a linear equation representing the data is

$$y = \frac{-8.7}{2{,}500}x + 100$$

This equation is an example of a mathematical *model*. A mathematical model is a mathematical object like an equation, a function, or a geometric figure that we use to represent a real-life situation. Sometimes a situation can be modeled by a linear function. We have to use judgment about whether this is a reasonable thing to do based on the information we are given. We must also be aware that the model may make imprecise predictions, or may only be appropriate for certain ranges of values.

Testing our model for the boiling point of water, it accurately predicts that at an elevation of 1,000 m above sea level (when $x = 1{,}000$), water will boil at 96.5° C since $y = \frac{-8.7}{2500} \cdot 1000 + 100 = 96.5$. For higher elevations, the model is not as accurate, but it is still close. At 5,000 m above sea level, it predicts 82.6° C, which is 0.6° C off the actual value of 83.2° C. At 9,000 m above sea level, it predicts 68.7° C, which is about 3° C less than the actual value of 71.5° C. The model continues to be less accurate at even higher elevations since the relationship between the boiling point of water and elevation isn't linear, but for the elevations in which most people live, it's pretty good.

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Unit 5, Lesson 9: Linear Models

1. On the first day after the new moon, 2% of the moon's surface is illuminated. On the second day, 6% is illuminated.

    a. Based on this information, predict the day on which the moon's surface is 50% illuminated and 100% illuminated.

    b. The moon's surface is 100% illuminated on day 14. Does this agree with the prediction you made?

    c. Is the percentage illumination of the moon's surface a linear function of the day?

2. In science class, Jada uses a graduated cylinder with water in it to measure the volume of some marbles. After dropping in 4 marbles so they are all under water, the water in the cylinder is at a height of 10 milliliters. After dropping in 6 marbles so they are all under water, the water in the cylinder is at a height of 11 milliliters.

    a. What is the volume of 1 marble?

    b. How much water was in the cylinder before any marbles were dropped in?

    c. What should be the height of the water after 13 marbles are dropped in?

    d. Is the volume of water a linear relationship with the number of marbles dropped in the graduated cylinder? If so, what does the slope of the line mean? If not, explain your reasoning.

3. Solve each of these equations. Explain or show your reasoning.

    $2(3x + 2) = 2x + 28$ $5y + 13 = -43 - 3y$ $4(2a + 2) = 8(2 - 3a)$

    (from Unit 4, Lesson 5)

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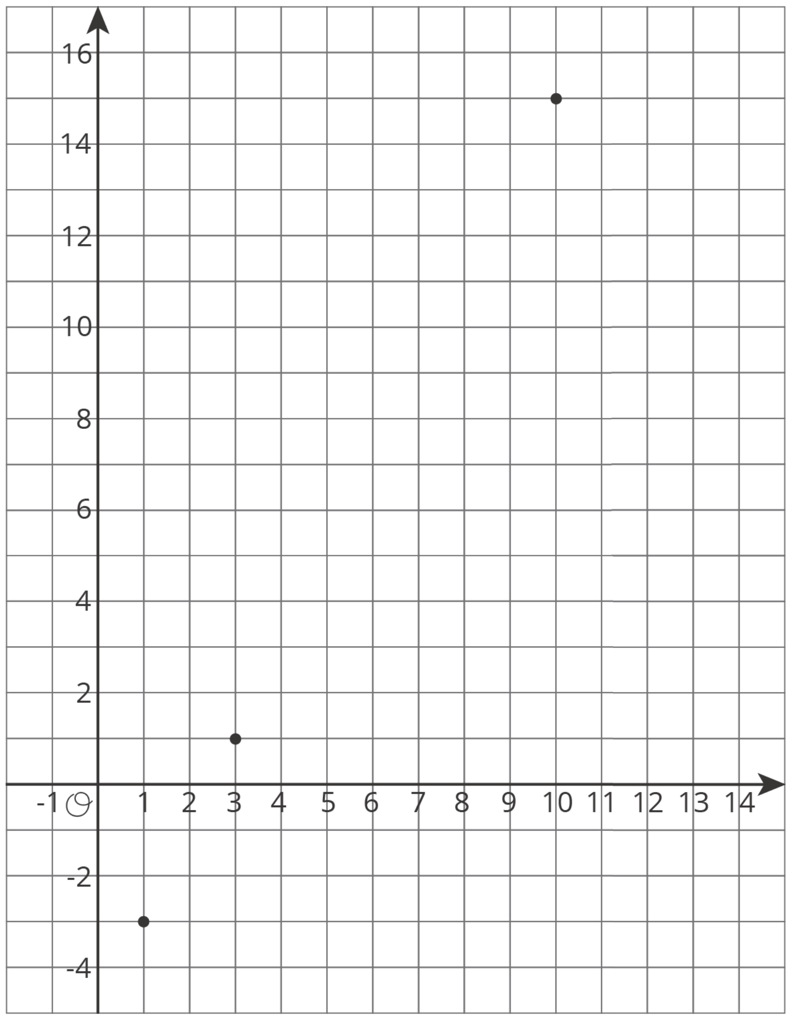
4. For a certain city, the high temperatures (in degrees Celsius) are plotted against the number of days after the new year.



Based on this information, is the high temperature in this city a linear function of the number of days after the new year?

5. The school designed their vegetable garden to have a perimeter of 32 feet with the length measuring two feet more than twice the width.

   a. Using ℓ to represent the length of the garden and w to represent its width, write and solve a system of equations that describes this situation.

   b. What are the dimensions of the garden?

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(from Unit 4, Lesson 15)