

NAME

DATE

PERIOD

Unit 4, Lesson 10: Dividing by Unit and Non-Unit Fractions

Let's look for patterns when we divide by a fraction.

10.1: Dividing by a Whole Number

Work with a partner. One person should solve the problems labeled "Partner A," and the other should solve those labeled "Partner B." Write an equation for each question. If you get stuck, draw a diagram.

1. Partner A

a. How many 3s are in 12?

Division equation:

b. How many 4s are in 12?

Division equation:

c. How many 6s are in 12?

Division equation:

2. Partner B

a. What is 12 groups of $\frac{1}{3}$?

Multiplication equation:

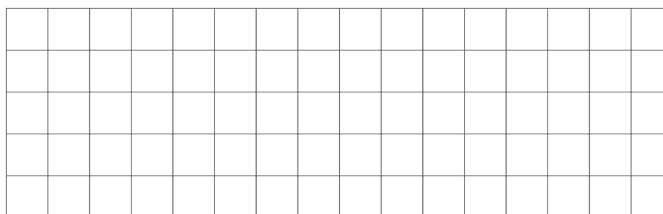
NAME _____

DATE _____

PERIOD _____

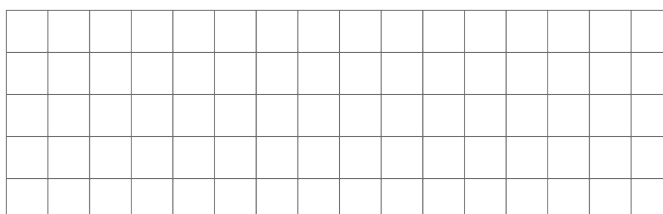
b. What is 12 groups of $\frac{1}{4}$?

Multiplication equation: _____



c. What is 12 groups of $\frac{1}{6}$?

Multiplication equation: _____

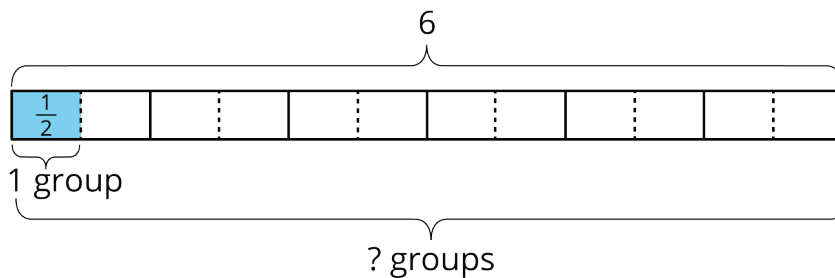


3. What do you notice in the diagrams and equations? Discuss with your partner.
4. Complete this sentence based on your observations: Dividing by a whole number a produces the same result as multiplying by _____ .

10.2: Dividing by Unit Fractions

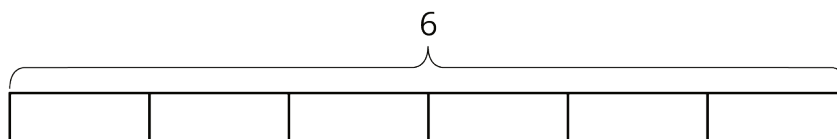
1. To find the value of $6 \div \frac{1}{2}$, Elena thought, "How many $\frac{1}{2}$ s are in 6?" and drew a tape diagram. It shows 6 ones with each one partitioned into 2 equal pieces.

$$6 \div \frac{1}{2}$$



For each division expression, complete the diagram using the same interpretation of division as Elena's. Then, write the value of the expression. Think about how to find that value without counting the pieces in the diagram.

a. $6 \div \frac{1}{3}$



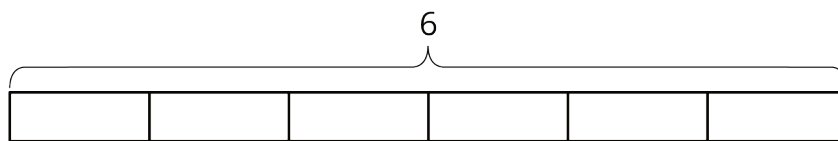
Value of the expression: _____

NAME _____

DATE _____

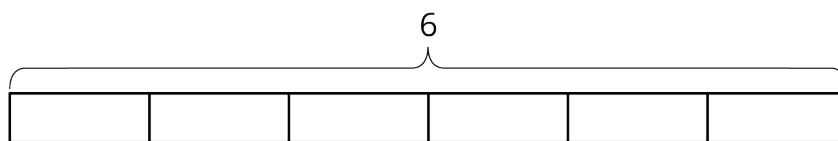
PERIOD _____

b. $6 \div \frac{1}{4}$



Value of the expression: _____

c. $6 \div \frac{1}{6}$



Value of the expression: _____

2. Analyze the expressions and your answers. Look for a pattern. How did you find how many $\frac{1}{2}$ s, $\frac{1}{3}$ s, $\frac{1}{4}$ s, or $\frac{1}{6}$ s were in 6 without counting? Explain your reasoning.

3. Use your observations from previous questions to find the values of the following expressions. If you get stuck, you can draw diagrams.

a. $6 \div \frac{1}{8}$

c. $6 \div \frac{1}{25}$

b. $6 \div \frac{1}{10}$

d. $6 \div \frac{1}{b}$

4. Find the value of each expression.

a. $8 \div \frac{1}{4}$

c. $a \div \frac{1}{2}$

b. $12 \div \frac{1}{5}$

d. $a \div \frac{1}{b}$

NAME _____

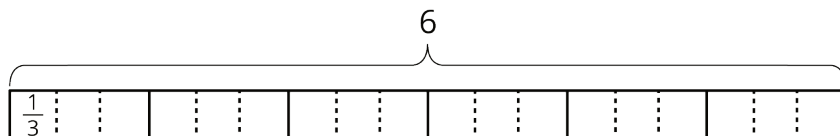
DATE _____

PERIOD _____

10.3: Dividing by Non-unit Fractions

1. To find the value of $6 \div \frac{2}{3}$, Elena began by drawing her diagram in the same way she did for $6 \div \frac{1}{3}$.

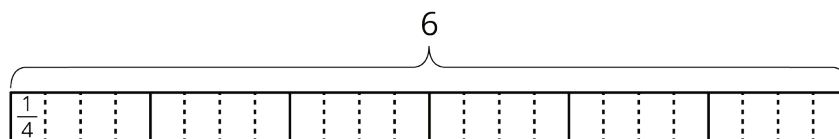
a. Use her diagram to find out how many $\frac{2}{3}$ s are in 6. Adjust and label the diagram as needed.



b. She says, "To find $6 \div \frac{2}{3}$, I can just take the value of $6 \div \frac{1}{3}$ then either multiply it by $\frac{1}{2}$ or divide it by 2." Do you agree with her? Explain why or why not.

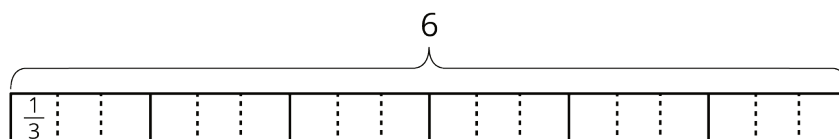
2. For each division expression, complete the diagram using the same interpretation of division that Elena did. Then, write the value of the expression. Think about how you could find the value of each expression without counting the equal pieces in your diagram.

$6 \div \frac{3}{4}$



Value of the expression: _____

$6 \div \frac{4}{3}$



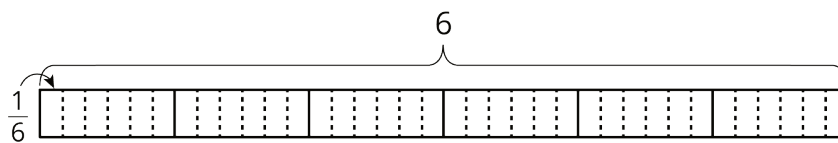
Value of the expression: _____

NAME _____

DATE _____

PERIOD _____

$$6 \div \frac{4}{6}$$



Value of the expression: _____

3. Elena studied her diagrams and noticed that she always took the same two steps to represent division by a fraction on a tape diagram. She said:

“My first step was to partition each 1 whole into as many parts as the number in the denominator. So if the expression is $6 \div \frac{3}{4}$, I would partition each 1 whole into 4 parts. Now I have 4 times as many parts.

My second step was to put a certain number of those parts into one group, and that number is the numerator of the divisor. So if the fraction is $\frac{3}{4}$, I would put 3 of the $\frac{1}{4}$ s into one group. I could then tell how many $\frac{3}{4}$ s are in 6.”

Which expression represents how many $\frac{3}{4}$ s Elena would have after these two steps? Be prepared to explain your reasoning.

a. $6 \div 4 \cdot 3$

c. $6 \cdot 4 \div 3$

b. $6 \div 4 \div 3$

d. $6 \cdot 4 \cdot 3$

4. Use your work from the previous questions to find the values of the following expressions. Draw diagrams if you are stuck.

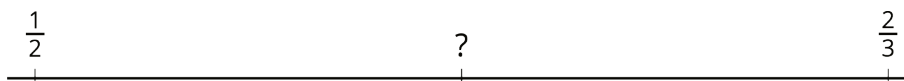
a. $6 \div \frac{2}{7}$

b. $6 \div \frac{3}{10}$

c. $6 \div \frac{6}{25}$

Are you ready for more?

Find the missing value.



NAME _____

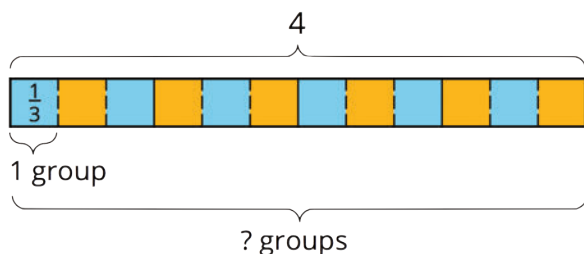
DATE _____

PERIOD _____

Lesson 10 Summary

To answer the question “How many $\frac{1}{3}$ s are in 4?” or “What is $4 \div \frac{1}{3}$?”, we can reason that there are 3 thirds in 1, so there are $(4 \cdot 3)$ thirds in 4.

In other words, dividing 4 by $\frac{1}{3}$ has the same outcome as multiplying 4 by 3.

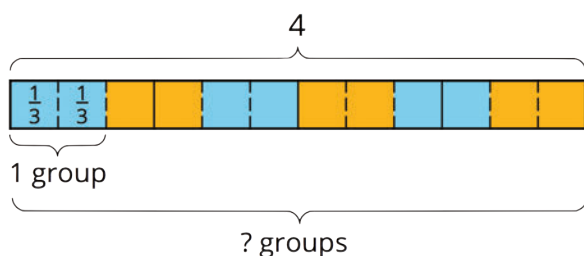


$$4 \div \frac{1}{3} = 4 \cdot 3$$

In general, dividing a number by a unit fraction $\frac{1}{b}$ is the same as multiplying the number by b , which is the reciprocal of $\frac{1}{b}$.

How can we reason about $4 \div \frac{2}{3}$?

We already know that there are $(4 \cdot 3)$ or 12 groups of $\frac{1}{3}$ s in 4. To find how many $\frac{2}{3}$ s are in 4, we need to put together every 2 of the $\frac{1}{3}$ s into a group. Doing this results in half as many groups, which is 6 groups. In other words:



$$4 \div \frac{2}{3} = (4 \cdot 3) \div 2$$

or

$$4 \div \frac{2}{3} = (4 \cdot 3) \cdot \frac{1}{2}$$

In general, dividing a number by $\frac{a}{b}$, is the same as multiplying the number by b and then dividing by a , or multiplying the number by b and then by $\frac{1}{a}$.

NAME _____

DATE _____

PERIOD _____

Unit 4, Lesson 10: Dividing by Unit and Non-Unit Fractions

1. Priya is sharing 24 apples equally with some friends. She uses division to determine how many people can have a share if each person gets a particular number of apples. For example, $24 \div 4 = 6$ means that if each person gets 4 apples, 6 people can have apples. Here are some other calculations:

$$24 \div 4 = 6$$

$$24 \div 2 = 12$$

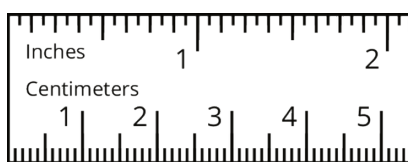
$$24 \div 1 = 24$$

$$24 \div \frac{1}{2} = ?$$

a. Priya thinks the “?” represents a number less than 24. Do you agree? Explain or show your reasoning.

b. In the case of $24 \div \frac{1}{2} = ?$, how many people can have apples?

2. Here is a centimeter ruler.



a. Use the ruler to find $1 \div \frac{1}{10}$ and $4 \div \frac{1}{10}$.

b. What calculation did you do each time?

c. Use your work from the first part to find each quotient.

i. $18 \div \frac{1}{10}$

ii. $4 \div \frac{2}{10}$

iii. $4 \div \frac{8}{10}$

3. Find each quotient.

a. $5 \div \frac{1}{10}$

b. $5 \div \frac{3}{10}$

c. $5 \div \frac{9}{10}$

NAME

DATE

PERIOD

4. Use the fact that $2\frac{1}{2} \div \frac{1}{8} = 20$ to find $2\frac{1}{2} \div \frac{5}{8}$. Explain or show your reasoning.

5. It takes one week for a crew of workers to pave $\frac{3}{5}$ kilometer of a road. At that rate, how long will it take to pave 1 kilometer?

Write a multiplication equation and a division equation that represent the question and then answer the question. Show your reasoning.

(from Unit 4, Lesson 9)

6. A box contains $1\frac{3}{4}$ pounds of pancake mix. Jada used $\frac{7}{8}$ pound for a recipe. What fraction of the pancake mix in the box did she use? Explain or show your reasoning. Draw a diagram, if needed.

(from Unit 4, Lesson 7)

7. Calculate each percentage mentally.

a. 25% of 400

c. 75% of 200

e. 5% of 20

b. 50% of 90

d. 10% of 8,000

(from Unit 3, Lesson 14)