GCE A level Mathematics (9AM0) – Paper 2
Pure Mathematics 2

October 2020 student-friendly mark scheme

Please note that this mark scheme is not the one used by examiners for making scripts. It is intended more as a guide to good practice, indicating where marks are given for correct answers. As such, it doesn't show follow-through marks (marks that are awarded despite errors being made) or special cases.

It should also be noted that for many questions, there may be alternative methods of finding correct solutions that are not shown here – they will be covered in the formal mark scheme.

This document is intended for guidance only and may differ significantly from the final mark scheme published in December 2020.

Guidance on the use of codes within this document
M1 – method mark. This mark is generally given for an appropriate method in the context of the question. This mark is given for showing your working and may be awarded even if working is incorrect. A1 – accuracy mark. This mark is generally given for a correct answer following correct working. B1 – working mark. This mark is usually given when working and the answer cannot easily be separated. Some questions require all working to be shown; in such questions, no marks will be given for an answer with no working (even if it is a correct answer).

Question 1 (Total 5 marks)

Part	Working or answer an examiner might expect to see	Mark	Notes
(a)	$h = 5$	B1 1.1a	This mark is given for stating $h = 5$
	$\frac{0.5}{2}$ {0.5774 + 0.8452 + 2(0.7071 + 0.7746 + 0.8165)}	M1 1.1b	This mark is given for a method to use the trapezium rule
	1.50	A1 1.1b	This mark is given for the correct answer only
(b)	$3 \times 1.50 = 4.50$	B1 2.2a	This mark is given for a correct estimate only
(c)	The answer is accurate to 2 significant figures	B1 3.2b	This mark is given for a correct statement about the accuracy

Question 2 (Total 3 marks)

Part	Working or answer an examiner might expect to see	Mark	Notes
	$\overrightarrow{PQ} = \mathbf{q} - \mathbf{p}$, $\overrightarrow{PR} = \mathbf{r} - \mathbf{p}$, $\overrightarrow{QR} = \mathbf{r} - \mathbf{q}$	M1 1.1b	This mark is given for a method to find at least two of the relevant vectors
	$\mathbf{r} - \mathbf{q} = 2(\mathbf{q} - \mathbf{p})$	M1 3.1a	This mark is given for using the information written in vector form
	$\mathbf{r} - \mathbf{q} = 2\mathbf{q} - 2\mathbf{p}$ $2\mathbf{p} + \mathbf{r} = 3\mathbf{q}$ $\mathbf{q} = \frac{1}{3}(\mathbf{r} + 2\mathbf{p})$	A1 2.1	The mark is given for fully correct working leading to the given answer

Question 3 (Total 5 marks)

Part	Working or answer an examiner might expect to see	Mark	Notes
(a)	$2 \log (4 - x) = \log (4 - x)^2$	B1 1.2	This mark is given for stating that $2 \log (4 - x) = \log (4 - x)^2$
	$2 \log (4 - x) = \log (x + 8)$ $\log (4 - x)^2 = \log (x + 8)$ $(4 - x)^2 = (x + 8)$	M1 1.1b	This mark is given for a method to eliminate logs to form a quadratic equation
	$16 + 8x + x^2 = x + 8$ $x^2 - 9x + 8 = 0$	A1 2.1	The mark is given for fully correct working leading to the given answer
(b)(i)	$(x - 1)(x - 8) = 0$ $x = 1, x = 8$	B1 1.1b	This mark is given for the correct answer only
(b)(ii)	8 is not a solution since log (4 – 8) cannot be found	B1 2.3	This mark is given for a correct reason stated

Question 4 (Total 3 marks)

Part	Working or answer an examiner might expect to see	Mark	Notes
	${}^7C_4\, a^3\, (2x)^4$	M1 1.1b	This mark is given for selecting the correct term
	$\frac{7!}{4!3!}\, a^3 \times 2^4 = 15120$	M1 2.1	This mark is given for a complete method to find the value of a
	$a = 3$	A1 1.1b	This mark is given for the correct answer only

Question 5 (Total 6 marks)

Part	Working or answer an examiner might expect to see	Mark	Notes
	$15 - 2^{x+1} = 3 \times 2^x$	B1 1.1b	This mark is given for finding $15 - 2^{x+1} = 3 \times 2^x$
	$15 - 2 \times 2^x = 3 \times 2^x$ $2^x = c$	M1 1.1b	This mark is given for a method to obtain an equation with 2^x as the subject
	$x = \log_2 c$	M1 1.1b	This mark is given for a full method to find the x-coordinate
	$x = \log_2 3$	A1 1.1b	This mark is given for the correct answer only

Question 6 (Total 7 marks)

Part	Working or answer an examiner might expect to see	Mark	Notes
(a)	$x^2 + 8x - 3 = (Ax + B)(x + 2) + C$	M1 1.1b	This mark is given for a method to find the values of A, B and C
	$\frac{x^2 + 8x - 3}{x + 2} = (x + 6) - 15$ $A = 1$, $B = 6$, $C = -15$	A1 1.1b	This mark is given for two of three correct values A, B and C
		A1 1.1b	This mark is given for a a third correct value A, B and C
(b)	$\int \frac{x^2 + 8x - 3}{x + 2} \, dx = \int x + 6 - \frac{15}{x + 2} \, dx$	M1 1.1b	This mark is given for integrating an expression of the form $\frac{C}{x + 2}$
	$= \frac{1}{2} x^2 + 6x - 15 \ln (x + 2) + c$	A1 1.1b	This mark is given for a correct integration of $\frac{A}{x + 2} + Bx + C$
	$\int_0^6 \frac{x^2 + 8x - 3}{x + 2} \, dx =$ $\left[\frac{1}{2}x^2 + 6x - 15\ln(x + 2)\right]_0^6$ $= (18 + 36 - 15 \ln 8) - (0 + 0 - 15 \ln 2)$ $= 18 + 36 - (15 - 45) \ln 2$	M1 2.1	This mark is given for a method to use limits

	$=54 - 30 \ln 2$	A1 1.1b	This mark is given for a correct answer only

Question 7 (Total 10 marks)

Part	Working or answer an examiner might expect to see	Mark	Notes
(a)	$\frac{\mathrm{d}(\ln x)}{\mathrm{d}x} = \frac{1}{x}$	B1 1.1a	This mark is given for the derivative of $\ln x$ seen
	$2 \times \frac{3}{2} x^{\frac{1}{2}} + \frac{1}{2} \times \frac{1}{2} x^{-\frac{1}{2}}$	M1 1.1b	This mark is given for a method to differentiate $\frac{4x^2 + x}{2\sqrt{x}}$
		A1 1.1b	This mark is given for a correct differentiation
	$\frac{\mathrm{d}y}{\mathrm{d}x} = 3\sqrt{x} + \frac{1}{4\sqrt{x}} - \frac{4}{x}$ $= \frac{12x^2 + x - 16\sqrt{x}}{4x\sqrt{x}}$	A1 2.1	The mark is given for fully correct working leading to the given answer
(b)	$12x^2 + x - 16\sqrt{x} = 0$	M1 1.1b	This mark is given for a method to set $\frac{\mathrm{d}y}{\mathrm{d}x} = 0$ and divide by $\sqrt{x}$
	$12 x^{\frac{3}{2}} = 16 - \sqrt{x}$	M1 1.1b	This mark is given for a method to make $x^{\frac{3}{2}}$ the subject
	$x^{\frac{3}{2}} = \frac{4}{3} - \frac{\sqrt{x}}{12} \Rightarrow x = \left(\frac{4}{3} - \frac{\sqrt{x}}{2}\right)^{\frac{2}{3}}$	A1 2.1	The mark is given for fully correct working leading to the given answer
(c)	$x_2 = \sqrt[3]{\left(\frac{4}{3} - \frac{\sqrt{2}}{12}\right)^2}$	M1 1.1b	This mark is given for a method to use the iterative formula with $x_1 = 2$
	$x_2 = 1.13894$	A1 1.1b	This mark is given for a correct value for x_2
	$x = 1.15650$	A1 2.2a	This mark is given for a correct value for the x-coordinate of P

Question 8 (Total 6 marks)

Part	Working or answer an examiner might expect to see	Mark	Notes
	$f(x) = 2x^3 + \frac{1}{2}ax^2 - 23x + c$	M1 1.1b	This mark is given for a method to integrate $f'(x)$
		A1 1.1b	This mark is given for a fully correct integration (including $+c$)
	$c = -12$	B1 2.2a	This mark is given for using the y-intercept to find the value of c
	$f(-4) = 0$ $\Rightarrow 2 \times (-4)^3 + \frac{1}{2}a(-4)^2 - 23(-4) - 12 = 0$	M1 3.1a	This mark is igvne for a method to find a by setting $f(-4) = 0$
	$a = 6$	M1 1.1b	This mark is given for a correct value for a
	$f(x) = 2x^3 + 3x^2 - 23x - 12$	A1 2.1	This mark is given for the correct answer only

Question 9 (Total 6 marks)

Part	Working or answer an examiner might expect to see	Mark	Notes
(a)	$t = 0, \theta = 18$ $18 = A - B$	M1 1.1b	This mark is given for method to use the model with $t = 0, \theta = 18$
	$t = 10, \theta = 44$ $44 = A - Be^{-0.7}$	M1 3.1b	This mark is given for method to use the model with $t = 10, \theta = 44$
	$A = 69.6, B = 18$	M1 3.1a	This mark is given for solving a pair of simultaneous equations to find A and B
	$\theta = 69.6 - 51.6e^{0.07t}$	A1 3.3	This mark is given for an equation of the model
(b)	The maximum temperature is 69.6 °C	B1 3.4	This mark is given for identifying the value of A as the boiling point of the model
	The model is not appropriate since 69.6 °C is much lower than 78 °C	B1 3.5a	This mark is given for a correct statement about the accuracy of the model

Question 10 (Total 8 marks)

Part	Working or answer an examiner might expect to see	Mark	Notes
(a)	$\cos 3A = \cos (2A + A)$ $= \cos 2A \cos A - \sin 2A \sin A$	M1 3.1a	This mark is given for a method to use the compound angle formula for $\cos (2A + A)$
	$= (2 \cos^2 A - 1) \cos A - (2 \sin A \cos A) \sin A$	M1 1.1b	This mark is given for a method to use double angle identities for $\cos 2A$ and $\sin 2A$
	$= (2 \cos^2 A - 1) \cos A - 2 \cos (1 - \cos^2 A)$	M1 2.1	This mark is given for a method to get all in terms in $\cos A$
	$= 4 \cos^3 - 3 \cos A$	A1 1.1b	The mark is given for fully correct working leading to the given answer
(b)	$1 - \cos 3x = \sin^2 x$ $\cos^2 x + 3 \cos x - 4 \cos^3 x = 0$	M1 1.1b	This mark is given for a method to get all in terms in $\cos x$
	$\cos x (4 \cos^2 x - \cos x - 3) = 0$ $\cos x (4 \cos x + 3)(\cos x - 1) = 0$	M1 3.1a	This mark is given for finding a quadratic equation to be solved
	$-90°, 0°, 90°, 139°$	A1 1.1b	This mark is given for any two correct answers
		A1 2.1	This mark is given for the other two correct answers

Question 11 (Total 7 marks)

Part	Working or answer an examiner might expect to see	Mark	Notes
(a)	$x = -4, y = -5$	B1 1.1b	This mark is given for one coordinate found
	$P(-4, -5)$	B1 2.2a	This mark is given for a correct answer only
(b)	$3x + 40 = -2(x + 4) - 5$	M1 1.1b	This mark is given for a method to solve the equation
	$x = -10.6$	A1 2.1	This mark is given for a correct answer only
(c)	$a > 2$	B1 2.2a	This mark is given for a deduction that $a > 2$
	$a = \frac{5}{4}$	M1 3.1a	This mark is given for finding the value of a using $P(4, 5)$ and O
	$\{a : a \leq 1.25\} \cup \{a : a > 2\}$	A1 2.5	This mark is given for a correct answer only

Question 12 (Total 11 marks)

Part	Working or answer an examiner might expect to see	Mark	Notes
(a)(i)	$\int y\frac{\mathrm{d}x}{\mathrm{d}t}\,\mathrm{d}t = \int 5\sin 2t \times 6\cos t\ \mathrm{d}t$	M1 1.2	This mark is given for a method to find $\int y\frac{\mathrm{d}x}{\mathrm{d}t}\,\mathrm{d}t$
	Using $\sin 2t = 2\sin t\cos t$	M1 1.1b	This mark is given for a method to use $\sin 2t = 2\sin t\cos t$
	Area $= \int_0^{\frac{\pi}{2}} 60\sin t\cos^2 t\ \mathrm{d}t$	A1 2.1	The mark is given for fully correct working leading to the given answer
(a)(ii)	$\int 60\sin t\cos^2 t\ \mathrm{d}t = -20\cos^3 t$	M1 1.1b	This mark is given for a method to integrate
		A1 1.1b	This mark is given for a correct integration
	Area $= \left[-20\cos^3 t\right]_0^{\frac{\pi}{2}} = 0 - (-20) = 20$	A1 2.1	The mark is given for fully correct working leading to the given answer
(b)	$5\sin 2t = 4.2$ $\sin 2t = \frac{4.2}{5}$	M1 3.4	This mark is given for a method to find values for t
	$t = 0.4986,\ t = 1.072$	A1 1.1b	This mark is given for correct values for t
	$t = 0.4986 \Rightarrow x = 2.869$ $t = 1.072 \Rightarrow x = 5.629$	M1 3.4	This mark is given for a method to find values of x for both values of t
		A1 1.1b	This mark is given for correct values for x
	Width of path = 2.40 metres	A1 3.2a	This mark is given for a correct answer only

Question 13 (Total 6 marks)

Part	Working or answer an examiner might expect to see	Mark	Notes
(a)	$k = e^2$	B1 2.2a	This mark is given for a correct deduction
(b)	$\frac{d}{dx}(g(x)) = \frac{(\ln x - 2) \times \frac{3}{x} - (3\ln x - 7) \times \frac{1}{x}}{(\ln x - 2)^2}$	M1 1.1b	This mark is given for a method to use the quotient rule to differentiate
	$= \frac{1}{x(\ln x - 2)^2}$	A1 2.1	This mark is given for a correct differentiation
	$x > 0$ and $(\ln x - 2)$ is squared, so $g'(x) > 0$	A1 2.4	This mark is given for a correct statement
(c)	$3 \ln x - 7 > 0$ and $\ln x - 2 < 0$	M1 3.1a	This mark is given for a method to solve either $3 \ln x - 7 > 0$ or $\ln x - 2 < 0$
	$0 < a < e^2$, $a > e^{\frac{7}{3}}$	A1 2.2a	This mark is given for a correct answer only

Question 14 (Total 7 marks)

Part	Working or answer an examiner might expect to see	Mark	Notes
(a)	$(x - r)^2 + (y - r)^2 = r^2$ or $x^2 + y^2 - 2rx - 2ry + r^2 = 0$	B1 2.2a	This mark is given for deducing an equation of C
	$x^2 + (12 - 2x)^2 - 2rx - 2r(12 - 2x) + r^2 = 0$	M1 1.1b	This mark is given for an attempt to solve $y = 12 - 2x$ and $x^2 + y^2 - 2rx - 2ry + r^2 = 0$
	$x^2 + 144 + 24x + 4x^2 - 2rx - 24r + 4rx + r^2 = 0$ $\Rightarrow 5x^2 + (2r - 48)x + (r^2 - 24r + 144) = 0$	A1 2.1	The mark is given for fully correct working leading to the given answer
(b)	$b^2 - 4ac = 0$ $a = 5$, $b = 2r - 48$, $c = r^2 - 24r + 144$	M1 3.1a	This mark is given for a method to find $b^2 - 4ac = 0$
	$(2r - 48)^2 - 4 \times 5 \times (r^2 - 24r + 144) = 0$	A1 1.1b	This mark is given for a correct equation in r
	$r^2 - 18r + 36 = 0$ $(r - 9)^2 - 81 + 36 = 0$	M1 1.1b	This mark is given for a method tosolve to find r
	$r = 9 \pm 3\sqrt{5}$	A1 1.1b	This mark is given for a correct answer only

Question 15 (Total 8 marks)

Part	Working or answer an examiner might expect to see	Mark	Notes
(a)	$S_n = a + ar + ar^2 + \ldots + ar^{n-1}$	B1 1.2	This mark is given for writing the sum of S_n
	$rS_n = ar + ar^2 + ar^3 \ldots + ar^n$	M1 2.1	This mark is given for a method to use the sum of rS_n
	$S_n - rS_n = a - ar^n$	A1 1.1b	This mark is given for subtracting rS_n from S_n
	$S_n(1-r) = a(1-r^n) \Rightarrow S_n = \dfrac{a(1-r^n)}{1-r}$	A1 2.1	The mark is given for fully correct working leading to the given answer
(b)	$\dfrac{a(1-r^{10})}{1-r} = 4 \times \dfrac{a(1-r^5)}{1-r}$ $(1-r^{10}) = 4(1-r^5)$	M1 3.1a	This mark is given for a method to find an equation in terms of r^{10} and r^5
	$r^{10} - 4r^5 + 3$	A1 1.1b	This mark is given for a correct equation in terms of r^{10} and r^5
	$(r^5 - 1)(r^5 - 3) = 0$	M1 2.1	This mark is given for rearranging to find a quadratic in r^5 to be solved
	$r = \sqrt[5]{3}$ only (reject $r = 1$)	A1 1.1b	This mark is given for the correct answer only

Question 16 (Total 4 marks)

Part	Working or answer an examiner might expect to see	Mark	Notes
	Natural numbers are $3k$, $3k + 1$, $3k + 2$	M1 3.1a	This mark is given for realising that all natural numbers can be written as $3k$, $3k + 1$ and $3k + 2$
	$(3k)^2 = 9k^2$ $(3k + 1)^2 = 9k^2 + 6k + 1$ $(3k + 2)^2 = 9k^2 + 12k + 4$	M1 1.1b	This mark is given for a method to find a squared expression for all three expressions for the natural numbers
	$(3k)^2 = 3 \times (3k^2)$ $(3k + 1)^2 = 3(3k^2 + 2k) + 1$ $(3k + 2)^2 = 3(3k^2 + 4k + 1) + 1$	A1 2.1	This mark is for showing algebraically that each case is either a multiple of three or a multiple of three, plus one

	Each case is either a multiple of three or a multiple of three, plus one	A1 2.4	This mark is given for a conclusion stated