CALCULUS AB
SECTION I, Part A
Time—55 minutes
Number of questions—28

A CALCULATOR MAY NOT BE USED ON THIS PART OF THE EXAM.

Directions: Solve each of the following problems, using the available space for scratch work. After examining the form of the choices, decide which is the best of the choices given and fill in the corresponding circle on the answer sheet. No credit will be given for anything written in the exam book. Do not spend too much time on any one problem.

In this exam:

(1) Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

(2) The inverse of a trigonometric function f may be indicated using the inverse function notation f^{-1} or with the prefix "arc" (e.g., $\sin^{-1} x = \arcsin x$).

1. $\int \left(5e^{2x} + \frac{1}{x}\right) dx =$

(A) $\frac{5}{2}e^{2x} + \frac{2}{x^2} + C$

(B) $\frac{5}{2}e^{2x} + \ln|x| + C$

(C) $5e^{2x} + \frac{2}{x^2} + C$

(D) $5e^{2x} + \ln|x| + C$

(E) $10e^{2x} - \frac{1}{x^2} + C$

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2. If $f(x) = \sqrt{x} + \frac{3}{\sqrt{x}}$, then $f'(4) =$

(A) $\frac{1}{16}$ (B) $\frac{5}{16}$ (C) 1 (D) $\frac{7}{2}$ (E) $\frac{49}{4}$

3. $\int x^2\left(x^3 + 5\right)^6\,dx =$

(A) $\frac{1}{3}\left(x^3 + 5\right)^6 + C$

(B) $\frac{1}{3}x^3\left(\frac{1}{4}x^4 + 5x\right)^6 + C$

(C) $\frac{1}{7}\left(x^3 + 5\right)^7 + C$

(D) $\frac{3}{7}x^2\left(x^3 + 5\right)^7 + C$

(E) $\frac{1}{21}\left(x^3 + 5\right)^7 + C$

x	0	25	30	50
$f(x)$	4	6	8	12

4. The values of a continuous function f for selected values of x are given in the table above. What is the value of the left Riemann sum approximation to $\int_0^{50} f(x)\,dx$ using the subintervals $[0, 25]$, $[25, 30]$, and $[30, 50]$?

(A) 290 (B) 360 (C) 380 (D) 390 (E) 430

$$f(x) = \begin{cases} x^2 \sin(\pi x) & \text{for } x < 2 \\ x^2 + cx - 18 & \text{for } x \geq 2 \end{cases}$$

5. Let f be the function defined above, where c is a constant. For what value of c, if any, is f continuous at $x = 2$?

(A) 2 (B) 7 (C) 9 (D) $4\pi - 4$ (E) There is no such value of c.

6. Which of the following is an antiderivative of $3\sec^2 x + 2$?

(A) $3\tan x$ (B) $3\tan x + 2x$ (C) $3\sec x + 2x$ (D) $\sec^3 x + 2x$ (E) $6\sec^2 x \tan x$

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7. If $f(x) = x^2 - 4$ and g is a differentiable function of x, what is the derivative of $f(g(x))$?

(A) $2g(x)$ (B) $2g'(x)$ (C) $2xg'(x)$ (D) $2g(x)g'(x)$ (E) $2g(x) - 4$

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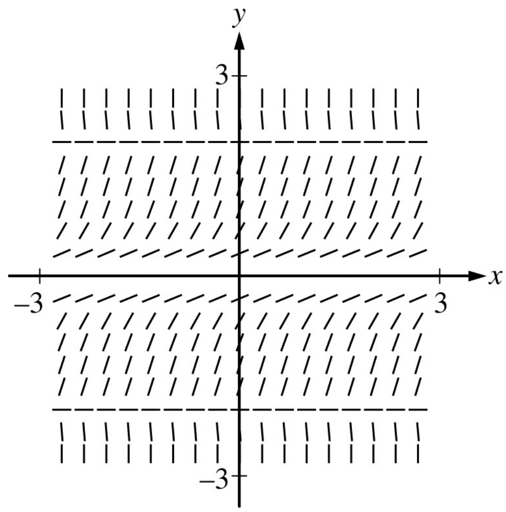

8. Shown above is a slope field for the differential equation $\frac{dy}{dx} = y^2\left(4 - y^2\right)$. If $y = g(x)$ is the solution to the differential equation with the initial condition $g(-2) = -1$, then $\lim_{x\to\infty} g(x)$ is

(A) $-\infty$ (B) -2 (C) 0 (D) 2 (E) 3

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9. If $f''(x) = x(x + 2)^2$, then the graph of f is concave up for

(A) $x < 0$

(B) $x > 0$

(C) $-2 < x < 0$

(D) $x < -2$ and $x > 0$

(E) all real numbers

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10. If $y = \sin x \cos x$, then at $x = \frac{\pi}{3}$, $\frac{dy}{dx} =$

(A) $-\frac{1}{2}$ (B) $-\frac{1}{4}$ (C) $\frac{1}{4}$ (D) $\frac{1}{2}$ (E) 1

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11. $\lim_{x \to -3} \frac{x^2 - 9}{x^2 - 2x - 15}$ is

(A) 0 (B) $\frac{3}{5}$ (C) $\frac{3}{4}$ (D) 1 (E) nonexistent

12. What is the average rate of change of $y = \cos(2x)$ on the interval $\left[0, \frac{\pi}{2}\right]$?

(A) $-\frac{4}{\pi}$ (B) -1 (C) 0 (D) $\frac{\sqrt{2}}{2}$ (E) $\frac{4}{\pi}$

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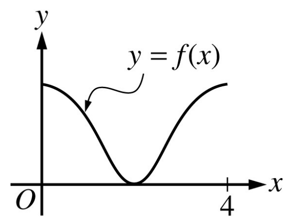
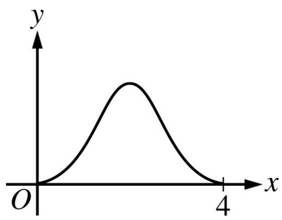
13. If $y^3 + y = x^2$, then $\frac{dy}{dx} =$

(A) 0 (B) $\frac{x}{2}$ (C) $\frac{2x}{3y^2}$ (D) $2x - 3y^2$ (E) $\frac{2x}{1 + 3y^2}$



14. The graph of $y = f(x)$ on the closed interval $[0, 4]$ is shown above. Which of the following could be the graph of $y = f'(x)$?

(A)



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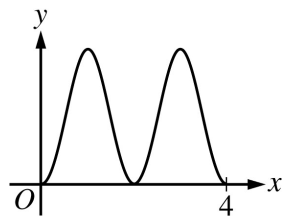

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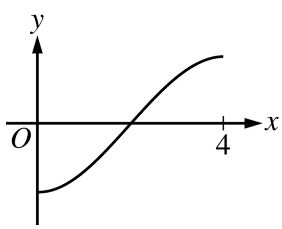

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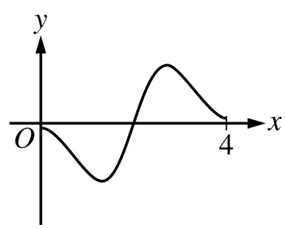
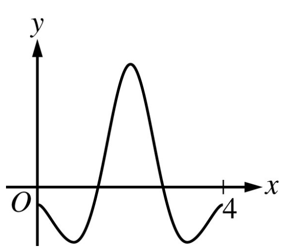


(E)



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$$f(x) = \begin{cases} 3x - 2 & \text{if } x < 1 \\ \ln(3x - 2) & \text{if } x \geq 1 \end{cases}$$

15. Let f be the function defined above. Which of the following statements about f are true?

I. $\lim_{x \to 1^-} f(x) = \lim_{x \to 1^+} f(x)$

II. $\lim_{x \to 1^-} f'(x) = \lim_{x \to 1^+} f'(x)$

III. f is differentiable at $x = 1$.

(A) None

(B) I only

(C) II only

(D) II and III only

(E) I, II, and III

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16. The function f is defined by $f(x) = 2x^3 - 4x^2 + 1$. The application of the Mean Value Theorem to f on the interval $1 \leq x \leq 3$ guarantees the existence of a value c, where $1 < c < 3$, such that $f'(c) =$

(A) 0 (B) 9 (C) 10 (D) 14 (E) 16

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17. The velocity v, in meters per second, of a certain type of wave is given by $v(h) = 3\sqrt{h}$, where h is the depth, in meters, of the water through which the wave moves. What is the rate of change, in meters per second per meter, of the velocity of the wave with respect to the depth of the water, when the depth is 2 meters?

(A) $-\frac{3}{4\sqrt{2}}$ (B) $-\frac{3}{8\sqrt{2}}$ (C) $\frac{3}{2\sqrt{2}}$ (D) $\frac{3}{\sqrt{2}}$ (E) $4\sqrt{2}$

18. If $\frac{dy}{dt} = -10e^{-t/2}$ and $y(0) = 20$, what is the value of $y(6)$?

(A) $20e^{-6}$ (B) $20e^{-3}$ (C) $20e^{-2}$ (D) $10e^{-3}$ (E) $5e^{-3}$

19. Let f be the function with derivative defined by $f'(x) = x^3 - 4x$. At which of the following values of x does the graph of f have a point of inflection?

(A) 0 (B) $\frac{2}{3}$ (C) $\frac{2}{\sqrt{3}}$ (D) $\frac{4}{3}$ (E) 2

20. Let f be the function given by $f(x) = \dfrac{(x-4)(2x-3)}{(x-1)^2}$. If the line $y = b$ is a horizontal asymptote to the graph of f, then $b =$

(A) 0 (B) 1 (C) 2 (D) 3 (E) 4

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21. The base of a solid is the region bounded by the x-axis and the graph of $y = \sqrt{1 - x^2}$. For the solid, each cross section perpendicular to the x-axis is a square. What is the volume of the solid?

(A) $\dfrac{2}{3}$ (B) $\dfrac{4}{3}$ (C) 2 (D) $\dfrac{2\pi}{3}$ (E) $\dfrac{4\pi}{3}$

22. Let f be the function given by $f(x) = \dfrac{kx}{x^2 + 1}$, where k is a constant. For what values of k, if any, is f strictly decreasing on the interval $(-1, 1)$?

(A) $k < 0$

(B) $k = 0$

(C) $k > 0$

(D) $k > 1$ only

(E) There are no such values of k.

23. Which of the following is an equation for the line tangent to the graph of $y = 3 - \int_{-1}^{x} e^{-t^3}\,dt$ at the point where $x = -1$?

(A) $y - 3 = -3e(x + 1)$

(B) $y - 3 = -e(x + 1)$

(C) $y - 3 = 0$

(D) $y - 3 = \frac{1}{e}(x + 1)$

(E) $y - 3 = 3e(x + 1)$

24. Which of the following is the solution to the differential equation $\frac{dy}{dx} = 5y^2$ with the initial condition $y(0) = 3$?

(A) $y = \sqrt{9e^{5x}}$

(B) $y = \sqrt{\frac{1}{9}e^{5x}}$

(C) $y = \sqrt{e^{5x} + 9}$

(D) $y = \frac{3}{1 - 15x}$

(E) $y = \frac{3}{1 + 15x}$

25. $\lim_{h \to 0} \dfrac{\sin\left(\frac{\pi}{3} + h\right) - \sin\left(\frac{\pi}{3}\right)}{h}$ is

(A) 0 (B) $\frac{1}{2}$ (C) 1 (D) $\frac{\sqrt{3}}{2}$ (E) nonexistent

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26. An object moves along a straight line so that at any time $t \geq 0$ its velocity is given by $v(t) = 2\cos(3t)$. What is the distance traveled by the object from $t = 0$ to the first time that it stops?

(A) 0 (B) $\frac{\pi}{6}$ (C) $\frac{2}{3}$ (D) $\frac{\pi}{3}$ (E) $\frac{4}{3}$

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x	$f(x)$	$f'(x)$
0	49	0
1	2	–8
2	–1	–80

27. The table above gives selected values for a differentiable and decreasing function f and its derivative. If f^{-1} is the inverse function of f, what is the value of $\left(f^{-1}\right)'(2)$?

(A) -80 (B) $-\frac{1}{8}$ (C) $-\frac{1}{80}$ (D) $\frac{1}{80}$ (E) $\frac{1}{8}$

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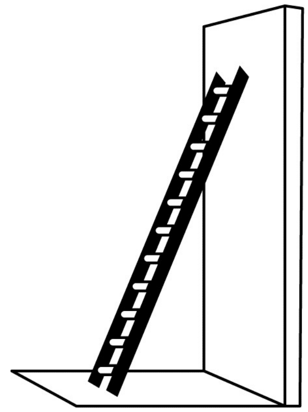

28. The top of a 15-foot-long ladder rests against a vertical wall with the bottom of the ladder on level ground, as shown above. The ladder is sliding down the wall at a constant rate of 2 feet per second. At what rate, in radians per second, is the acute angle between the bottom of the ladder and the ground changing at the instant the bottom of the ladder is 9 feet from the base of the wall?

(A) $-\frac{2}{9}$ (B) $-\frac{1}{6}$ (C) $-\frac{2}{25}$ (D) $\frac{2}{25}$ (E) $\frac{1}{9}$

END OF PART A OF SECTION I

IF YOU FINISH BEFORE TIME IS CALLED, YOU MAY CHECK YOUR WORK ON PART A ONLY.

DO NOT GO ON TO PART B UNTIL YOU ARE TOLD TO DO SO.