



AP[®] Calculus AB 2003 Free-Response Questions

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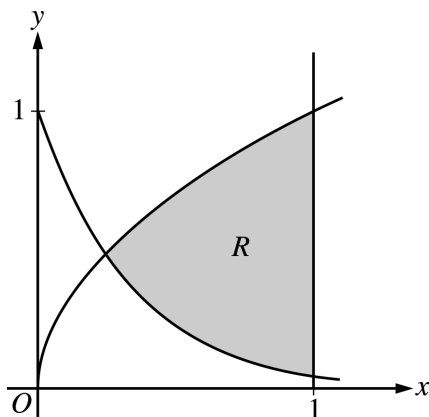
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2003 AP[®] CALCULUS AB FREE-RESPONSE QUESTIONS

CALCULUS AB
SECTION II, Part A
Time—45 minutes
Number of problems—3

A graphing calculator is required for some problems or parts of problems.



1. Let R be the shaded region bounded by the graphs of $y = \sqrt{x}$ and $y = e^{-3x}$ and the vertical line $x = 1$, as shown in the figure above.
- (a) Find the area of R .
- (b) Find the volume of the solid generated when R is revolved about the horizontal line $y = 1$.
- (c) The region R is the base of a solid. For this solid, each cross section perpendicular to the x -axis is a rectangle whose height is 5 times the length of its base in region R . Find the volume of this solid.
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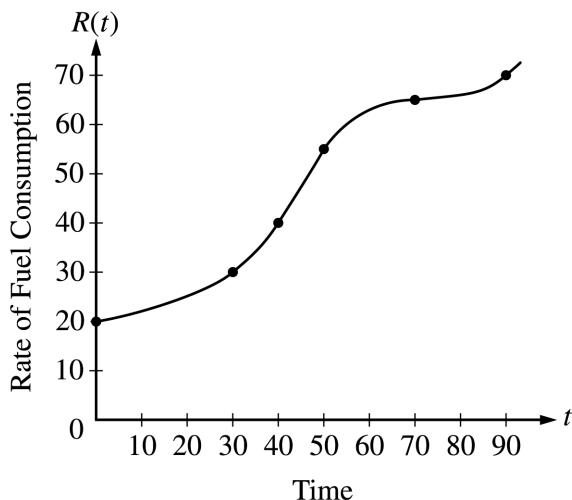
2. A particle moves along the x -axis so that its velocity at time t is given by

$$v(t) = -(t + 1) \sin\left(\frac{t^2}{2}\right).$$

At time $t = 0$, the particle is at position $x = 1$.

- (a) Find the acceleration of the particle at time $t = 2$. Is the speed of the particle increasing at $t = 2$? Why or why not?
 - (b) Find all times t in the open interval $0 < t < 3$ when the particle changes direction. Justify your answer.
 - (c) Find the total distance traveled by the particle from time $t = 0$ until time $t = 3$.
 - (d) During the time interval $0 \leq t \leq 3$, what is the greatest distance between the particle and the origin? Show the work that leads to your answer.
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t (minutes)	$R(t)$ (gallons per minute)
0	20
30	30
40	40
50	55
70	65
90	70

3. The rate of fuel consumption, in gallons per minute, recorded during an airplane flight is given by a twice-differentiable and strictly increasing function R of time t . The graph of R and a table of selected values of $R(t)$, for the time interval $0 \leq t \leq 90$ minutes, are shown above.
- (a) Use data from the table to find an approximation for $R'(45)$. Show the computations that lead to your answer. Indicate units of measure.
- (b) The rate of fuel consumption is increasing fastest at time $t = 45$ minutes. What is the value of $R''(45)$? Explain your reasoning.
- (c) Approximate the value of $\int_0^{90} R(t) dt$ using a left Riemann sum with the five subintervals indicated by the data in the table. Is this numerical approximation less than the value of $\int_0^{90} R(t) dt$? Explain your reasoning.
- (d) For $0 < b \leq 90$ minutes, explain the meaning of $\int_0^b R(t) dt$ in terms of fuel consumption for the plane. Explain the meaning of $\frac{1}{b} \int_0^b R(t) dt$ in terms of fuel consumption for the plane. Indicate units of measure in both answers.

END OF PART A OF SECTION II

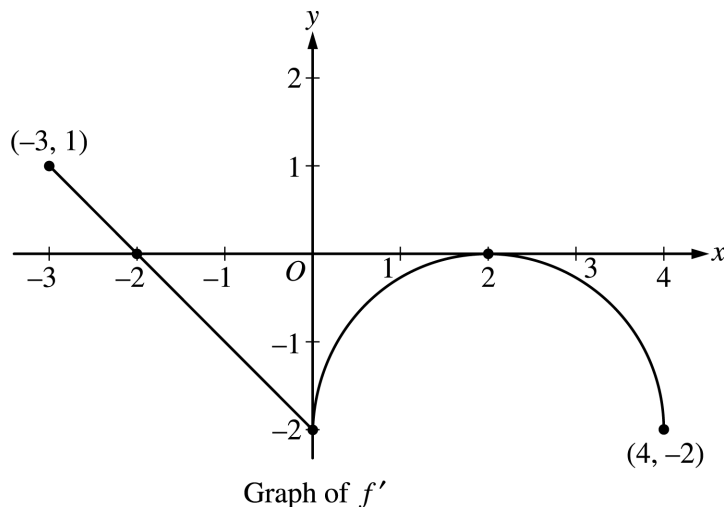
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CALCULUS AB
SECTION II, Part B

Time—45 minutes

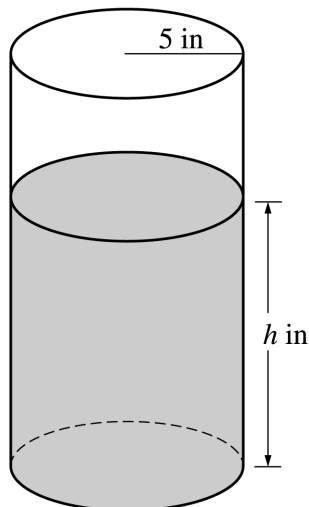
Number of problems—3

No calculator is allowed for these problems.



4. Let f be a function defined on the closed interval $-3 \leq x \leq 4$ with $f(0) = 3$. The graph of f' , the derivative of f , consists of one line segment and a semicircle, as shown above.
- On what intervals, if any, is f increasing? Justify your answer.
 - Find the x -coordinate of each point of inflection of the graph of f on the open interval $-3 < x < 4$. Justify your answer.
 - Find an equation for the line tangent to the graph of f at the point $(0, 3)$.
 - Find $f(-3)$ and $f(4)$. Show the work that leads to your answers.
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5. A coffeepot has the shape of a cylinder with radius 5 inches, as shown in the figure above. Let h be the depth of the coffee in the pot, measured in inches, where h is a function of time t , measured in seconds. The volume V of coffee in the pot is changing at the rate of $-5\pi\sqrt{h}$ cubic inches per second. (The volume V of a cylinder with radius r and height h is $V = \pi r^2 h$.)

(a) Show that $\frac{dh}{dt} = -\frac{\sqrt{h}}{5}$.

(b) Given that $h = 17$ at time $t = 0$, solve the differential equation $\frac{dh}{dt} = -\frac{\sqrt{h}}{5}$ for h as a function of t .

- (c) At what time t is the coffeepot empty?

6. Let f be the function defined by

$$f(x) = \begin{cases} \sqrt{x+1} & \text{for } 0 \leq x \leq 3 \\ 5-x & \text{for } 3 < x \leq 5. \end{cases}$$

- (a) Is f continuous at $x = 3$? Explain why or why not.
 (b) Find the average value of $f(x)$ on the closed interval $0 \leq x \leq 5$.
 (c) Suppose the function g is defined by

$$g(x) = \begin{cases} k\sqrt{x+1} & \text{for } 0 \leq x \leq 3 \\ mx+2 & \text{for } 3 < x \leq 5, \end{cases}$$

where k and m are constants. If g is differentiable at $x = 3$, what are the values of k and m ?

END OF EXAMINATION